

## Calculation Policy for Mathematics

### School Vision, Values and Aims

Our school vision, values and aims are rooted in the Church of England's Vision for Education; *Deeply Christian, Serving the Common Good* and the biblical teaching of;

*“Love the Lord your God with all your heart, and with all your soul, and with all your mind and with all your strength; and love your neighbour as yourself” (Mark 12.30,31).*

### Vision

Open Hearts, Open Minds, Open Doors

### Values

Respect	Luke 10:25-37	The Good Samaritan
Responsibility	Luke 15:11-32	The Prodigal Son
Compassion	Luke 19, 1-10	Zacchaeus the tax collector
Perseverance	Luke 5:17-26	A man is helped by friends to see Jesus

### Aims

Our principle aim - For all to know that they are welcome

To show kindness and forgiveness

To listen to one another

To seek enjoyment in our work

To show patience with one another

To show love towards one another

To show courage in our work

To keep the children at the heart of our decisions

To embrace innovation and change

To trust one another

To inspire one another

To celebrate the dignity and worth of each individual

To begin each new day positively

To reach out to one another

To have confidence

To encourage one another

To support and challenge one another

To believe in one another

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

It has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. The policy provides guidance on appropriate calculation methods and progression from EYFS to Year 6. The content is set out in yearly blocks and outlines a clear, smooth progression of learning calculations across the school.

### **Age- Related Expectations**

The calculation policy is organised according to age-related expectations as set out in the National Curriculum 2014. It is vital **that pupils are taught according to the stage that they are currently working at.**

Our overall aim is that when children leave Leckhampton they will:

- ❖ have a secure knowledge of number facts and a good understanding of the four operations;
- ❖ make use of diagrams and informal notes (jottings) to help record steps when using mental methods or as a visual representation of a mental method;
- ❖ have an efficient, reliable, formal, written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally.
- ❖ have a secure knowledge of multiplication facts and recall

This policy supports the White Rose maths scheme used throughout the school. It outlines the skills, representations and models for teaching and learning of calculation strategies in mathematics.

It is designed to help teachers and staff at Leckhampton ensure that calculation is taught consistently across the school to support children to develop a deep understanding of number and calculation. It is also designed to help parents, carers and other family members support children's learning by providing explanations of the representations and models used in our school.

Within each year group section there is information which shows the skills that will be taught and information on the models and images that could be used to support the teaching of that concept. This document also includes an overview of models and representations used, explanations of the benefits of using the models and highlights the links between the different key operations.

A glossary of terms is provided at the end of the calculation policy to support understanding of the key language used to teach the four key operations.

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**Reception**

**Addition**


- Have a deep understanding of numbers to 10, including the composition of each number.
- Subitise (recognise quantities without counting) up to 5
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.

**Key skill**      **Key representations**

**Conceptually subitise to 5**

What do you see?  
How do you see it?

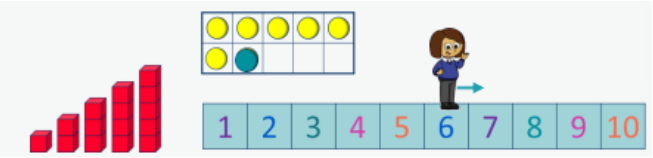
Notice the parts that make up the whole.



**1 more**

1 more than ... is ...

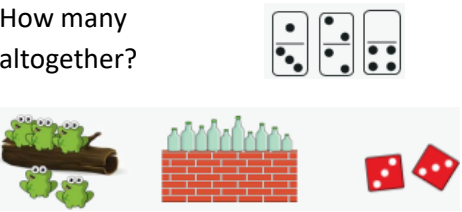
Continue to link to stories, songs and rhymes



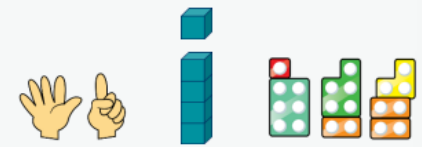
**Notice the composition of numbers within 10**

How many...?  
How many...?  
How many altogether?

Link to stories, songs and rhymes



How many ways can you make...?



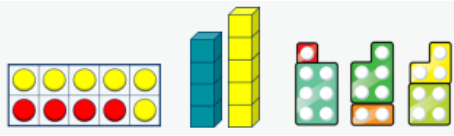
**Combine 2 groups**

2 groups are combined to find the total.

There are ...  
There are ...  
There are ... altogether.



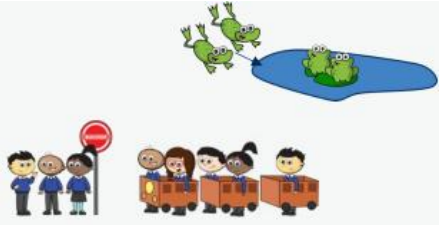
... and ... make ...



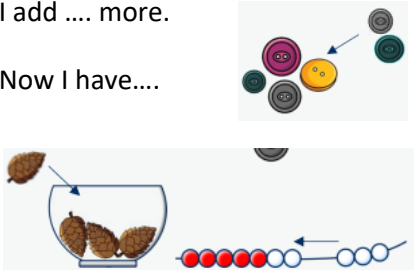
**Add more**

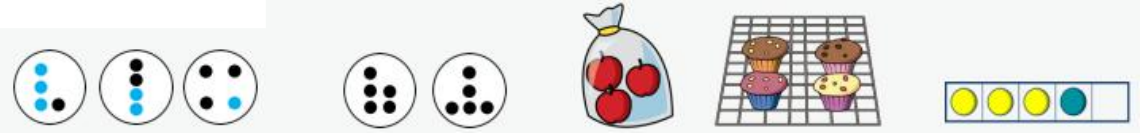
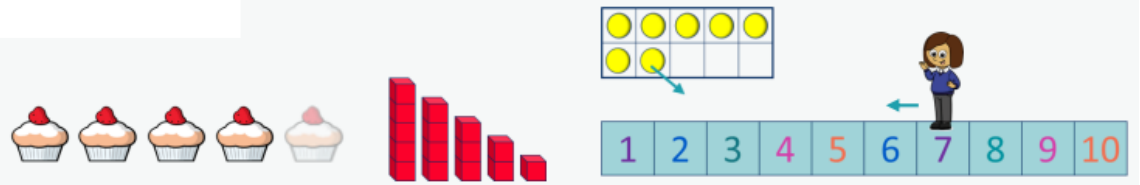
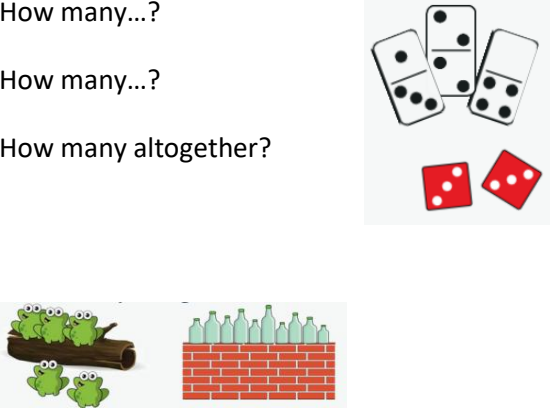
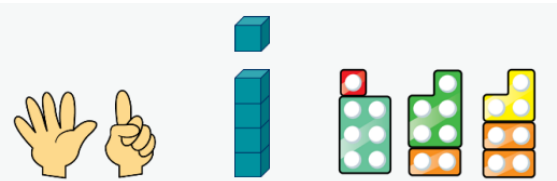

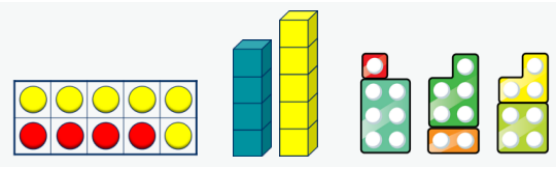
A quantity is increased.

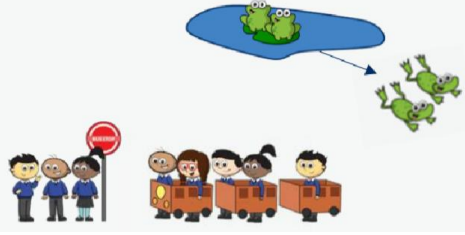


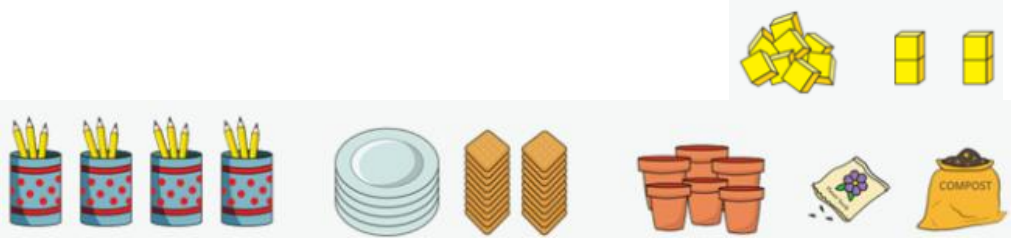
First... Then.... Now....



I have ...  
I add ... more.  
Now I have....



<p><b>Subtraction</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (and some subtraction facts) and some number bonds to 10, including double facts.</li> </ul>	
<p><b>Key skill</b></p>	<p><b>Key representations</b></p>	
<p><b>Conceptually subitise to 5</b></p> <p>Notice the parts that make up the whole.</p>	<p>What do you see?</p> <p>How do you see it?</p> 	
<p><b>1 less</b></p> <p>Continue to link to stories, songs and rhymes.</p>	<p>1 less than ... is ...</p> 	
<p><b>Notice the composition of numbers within 10</b></p> <p>Link to stories, songs and rhymes.</p>	<p>How many...?</p> <p>How many...?</p> <p>How many altogether?</p> 	<p>How many ways can you make...?</p> 
<p><b>Partition</b></p> <p>Using objects, explore different ways to partition a number into 2 or more parts.</p>	<p>There are ... altogether.</p> <p>I can see ... here and ... there.</p> 	<p>... and ... make ...</p> 
<p><b>Take away</b></p>	<p>First... Then... Now...</p>	<p>I have ...</p>

<p>A quantity is reduced.</p>		<p>I take ... away</p> <p>Now I have ...</p> 
<p><b>Multiplication</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> <li>• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li> </ul>	
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Double to 10</b></p> <p>Prompt children to notice that double means twice as many and to notice that there are two equal groups.</p>	<p>Double ... is ...</p> <p>... is double ...</p> 	
<p><b>Make equal groups</b></p> <p>Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.</p>	<p>There are ... groups of ...</p> <p>There are ... altogether.</p> 	
<p><b>Division</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> <li>• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li> </ul>	
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Sharing</b></p> <p>Provide practical activities such as sharing items during snack time. Encourage</p>	<p>There are ... altogether.</p> <p>They are shared equally between ... groups.</p>	

children to check whether items have been shared fairly (equally).



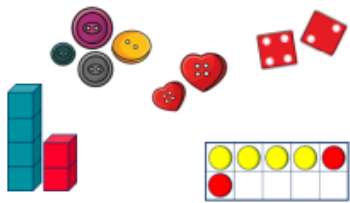
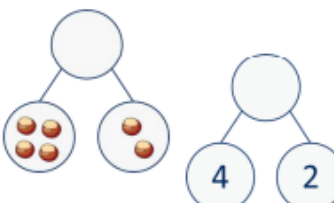
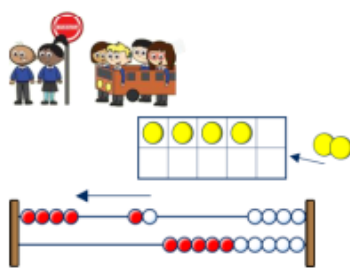
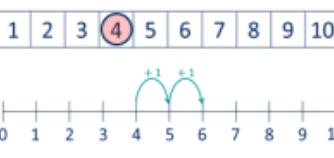
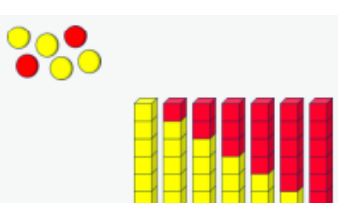
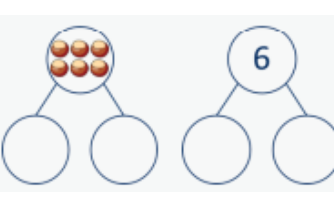

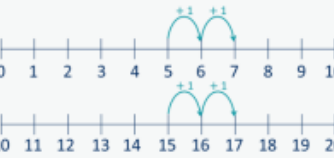
**Grouping**  
Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.

There are ... groups of ...

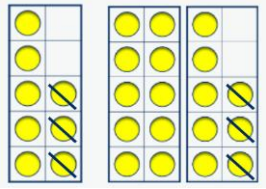
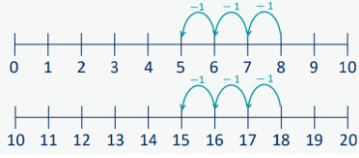
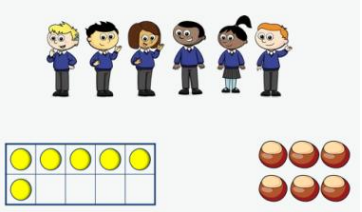
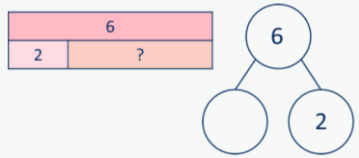

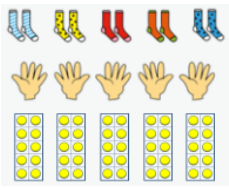

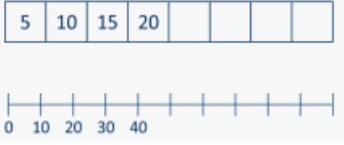
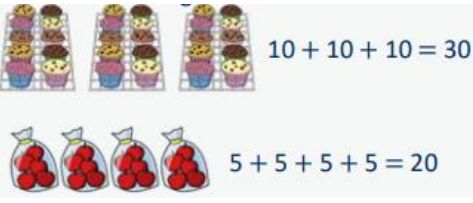
There are ... altogether.



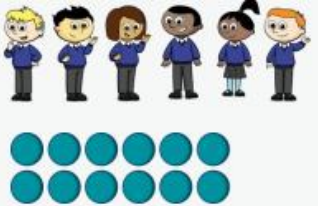
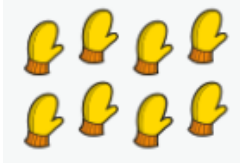
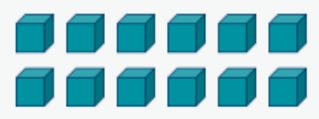
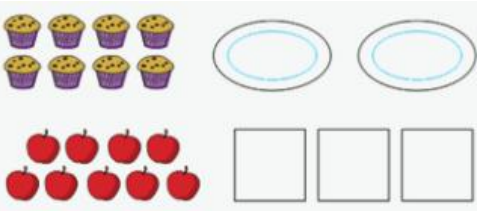



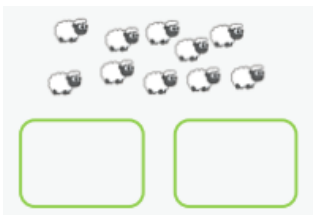
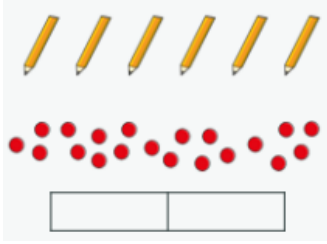
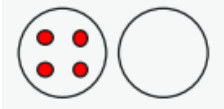
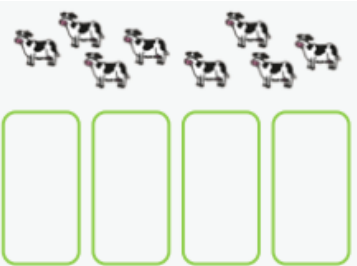
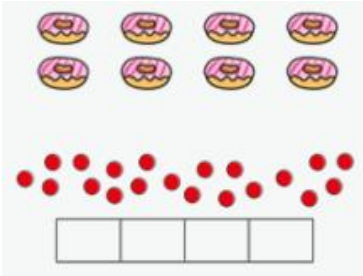
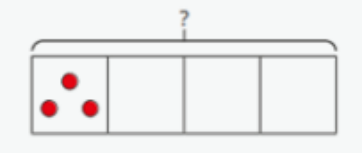
**Year 1**

<p><b>Addition</b></p>	<ul style="list-style-type: none"> <li>• Read, write and interpret mathematical statements involving addition (+) and equals (=) signs.</li> <li>• Represent and use number bonds within 20</li> <li>• Add 1-digit and 2-digit numbers to 20, including zero.</li> <li>• Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as <math>7 = \_\_\_ + 2</math></li> </ul>		
<p><b>Key skill</b></p>	<p><b>Key representations</b></p>		
<p><b>Add together</b> (aggregation)</p> <p>2 quantities are combined to find the total.</p>	<p>There are ... There are ... There are ... altogether.</p> 	<p>... is a part. ... is a part. ... is the whole.</p> 	<p>... plus ... is equal to ... ... is equal to ... + ...</p> $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$ $6 = 2 + 4$
<p><b>Add more</b> (augmentation)</p> <p>A quantity is increased.</p>	<p>First... Then... Now...</p> 	<p>I start at ... I jump on ... I land on ...</p> 	<p>... plus ... is equal to ... ... is equal to ... + ...</p> $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$ $6 = 2 + 4$
<p><b>Bonds within 10</b></p> <p>Include bonds for each number within 10</p> <p>Encourage children to notice patterns</p>	<p>... is made of ... and ... ... and ... make ...</p> 	<p>... can be partitioned into ... and ...</p> 	<p>... plus ... is equal to ...</p> $6 + 0 = 6$ $5 + 1 = 6$ $4 + 2 = 6$ $3 + 3 = 6$ $2 + 4 = 6$ $1 + 5 = 6$ $0 + 6 = 6$
<p><b>Related facts within 20</b></p> <p>Make links to known facts.</p>	<p>I know that ... and ... = ... so ... and ... = ...</p> 	<p>... more than ... is ... so ... more than ... is ...</p> 	<p>What patterns do you notice?</p> $5 + 2 = 7$ $15 + 2 = 17$ $7 = 5 + 2$ $17 = 15 + 2$

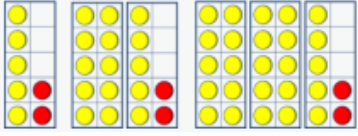
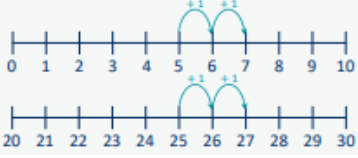
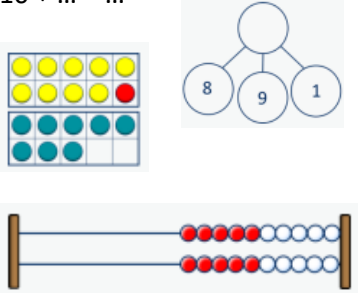
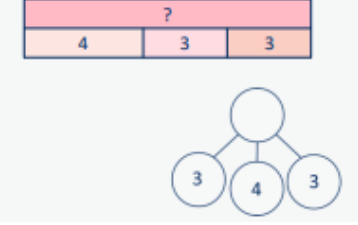
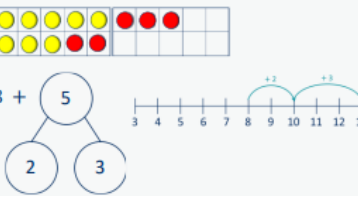
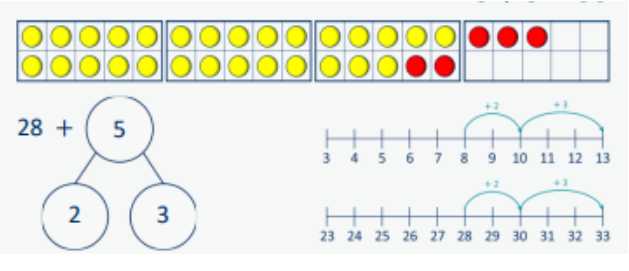


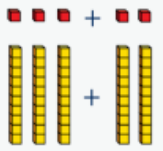
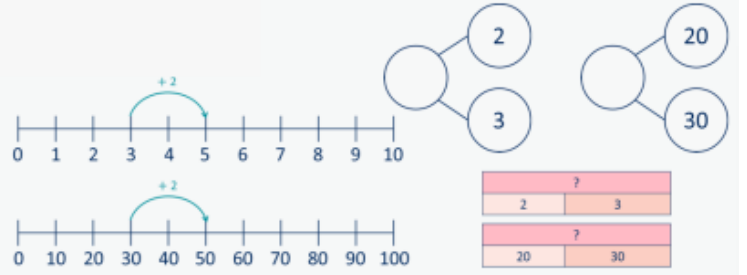
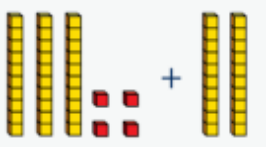
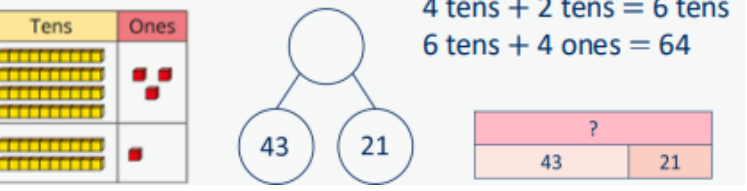
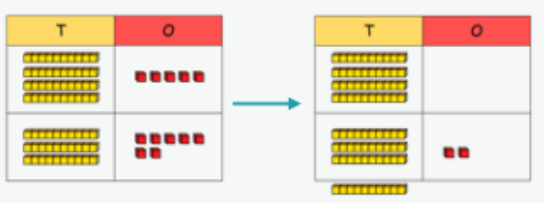
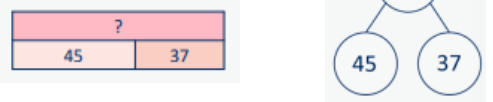
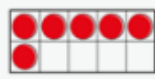
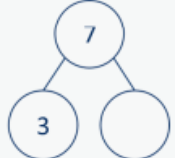
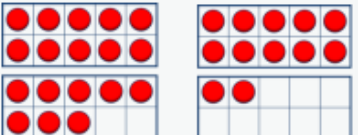
			$6 - 4 = 2$ $6 - 5 = 1$ $6 - 6 = 0$
<b>Related facts within 20</b>  Make links to known facts.	I know that ... minus ... = ... so ... minus ... = ...  	... less than ... is ... so ... less than ... is ...  	What patterns do you notice?  $8 - 3 = 5$ $18 - 3 = 15$ $5 = 8 - 3$ $15 = 18 - 3$
<b>Missing numbers</b>  Make links to known facts.	How many do you need to subtract to make ...?  	If ... is the whole and ... is a part, the other part must be...  	... minus ... is equal to ...  $6 - \square = 2$ $2 = 6 - \square$  
<b>Multiplication</b>	<ul style="list-style-type: none"> <li>Count in multiples of twos, fives and tens.</li> <li>Solve one-step problems involving multiplication, using concrete objects, pictorial representations and arrays with the support of the teacher.</li> </ul>		
<b>Key skills</b>	<b>Key representations</b>		
<b>Count in 2s, 5s and 10s</b>  Begin by counting objects that naturally come in 2s, 5s and 10s, for example pairs of socks or fingers.	There are ... equal groups of ... There are ... altogether.  	Continue to colour in ...s What do you notice?  	Complete the number track/number line by counting in ...s.  
<b>Add equal groups (repeated addition)</b>  Children should be able to write a repeated addition to represent equal groups and to draw pictures or use objects	There are ... groups of ... There are ... altogether.   $10 + 10 + 10 = 30$ $5 + 5 + 5 + 5 = 20$	What is the same? What is different?  $2 + 2 + 2 =$ $5 + 5 + 5 =$ $10 + 10 + 10 =$  Use objects or a drawing to represent the equal groups and find how many in total.	


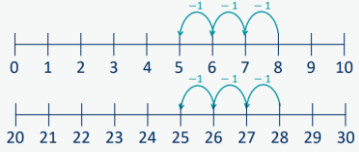
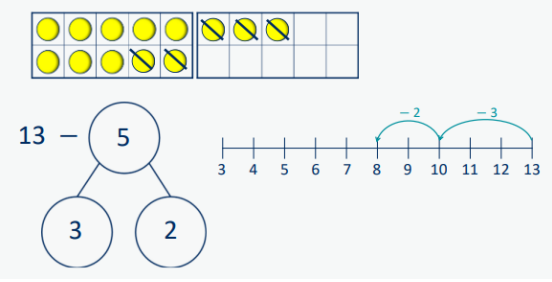
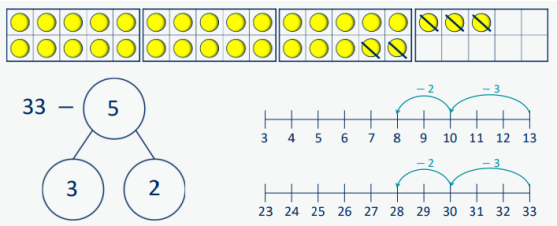
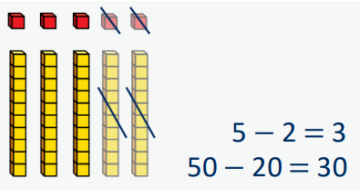
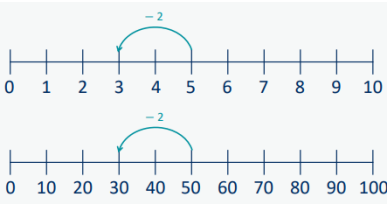

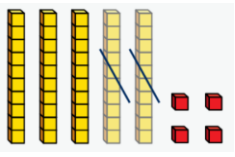
to represent a repeated addition.			
<p><b>Make arrays</b></p> <p>Children use their knowledge of adding equal groups to arrange objects in columns and rows.</p>	<p>There are ... rows of ... There are ... altogether. There are ... columns of ... There are ... altogether.</p> 		
<p><b>Make doubles</b></p> <p>Children understand that doubles are two equal groups. Children may begin to explore doubles beyond 20 using base 10</p>	<p>Double ... is ... ... + ... = ...</p> 		
<b>Division</b>	<ul style="list-style-type: none"> <li>• Solve simple one-step problems involving division, using concrete objects, pictorial representations and arrays with the support of the teacher.</li> <li>• Recognise, find and name a half as one of two equal parts of a quantity.</li> <li>• Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</li> </ul>		
<b>Key skills</b>	<b>Key representations</b>		
<p><b>Make equal groups – grouping</b></p> <p>Encourage children to physically move objects into equal groups. They can also circle equal groups when using pictures.</p>	<p>There are ... altogether. How many groups of ... can you make?</p> 	<p>Circle groups of 2 There are ... groups of 2</p> 	<p>Take ... cubes. Make equal groups. There are ... groups of ...</p> 
<p><b>Make equal groups – sharing</b></p> <p>Encourage children to check that the objects have been shared fairly and each group is the same.</p>	<p>... have been shared equally between... There are ... on/in each ...</p>  <p>Take ... cubes. Share them between ...</p>  <p>12 shared between ... is ...</p>		

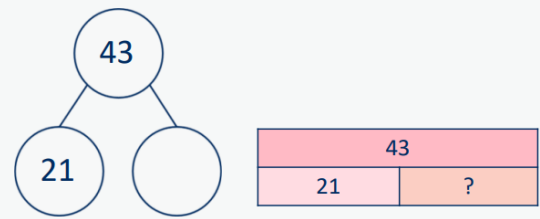
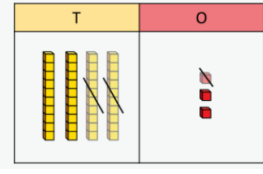
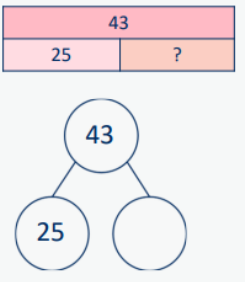
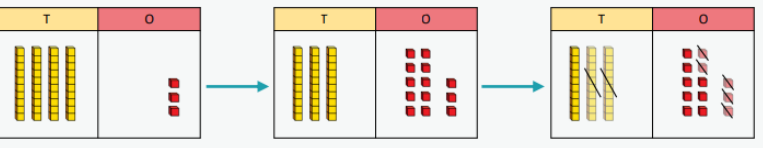
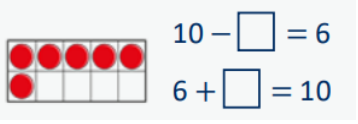
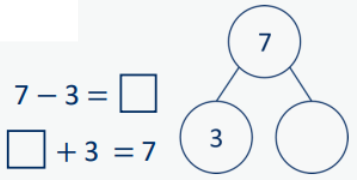
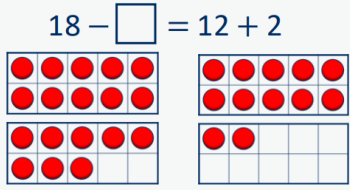
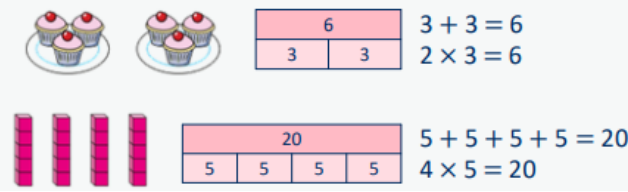
<p><b>Find a half</b></p> <p>Start with practical opportunities to share a quantity into 2 groups. Progress to circling half of the objects in a picture and then to finding the whole from a given half.</p>	<p>To find half, I need to share into 2 equal groups. There are ... in each group.</p> 	<p>Half of ... is ...</p> 	<p>If ... is half, what is the whole?</p>  <p>4 is half of ...</p>
<p><b>Find a quarter</b></p> <p>Start with practical opportunities to share a quantity into 4 groups. Progress to using pictures or bar models to find a quarter and then to finding the whole from a given quarter.</p>	<p>To find a quarter, I need to share into 4 equal groups. There are ... in each group.</p> 	<p>A quarter of ... is ...</p> 	<p>If ... is one quarter, what is the whole?</p>  <p>3 is one quarter of ...</p>



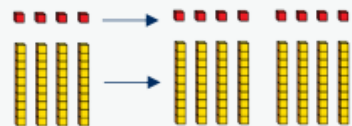

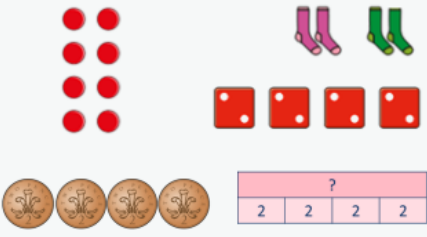


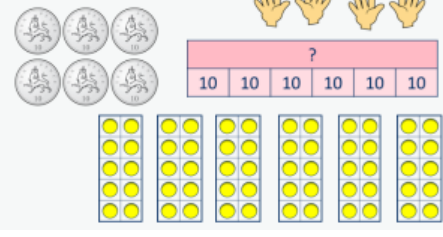
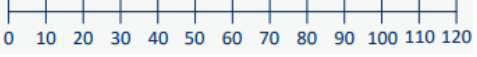

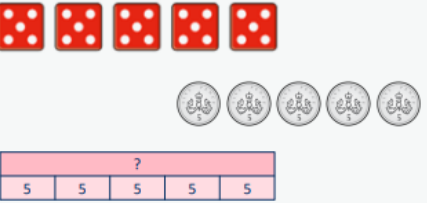

**Year 2**


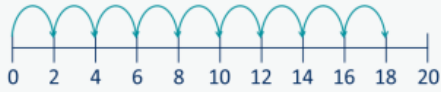
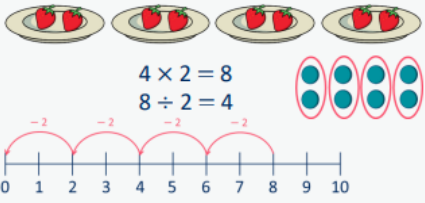
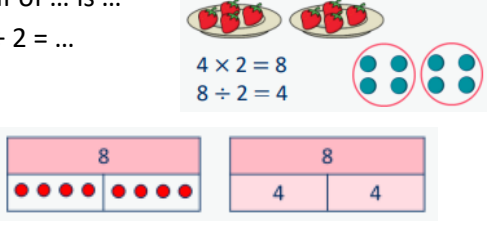
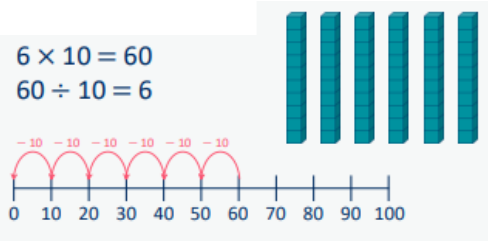
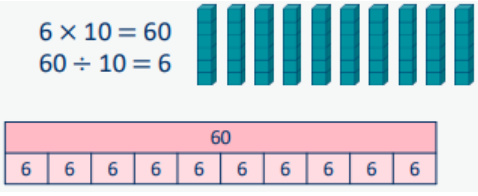
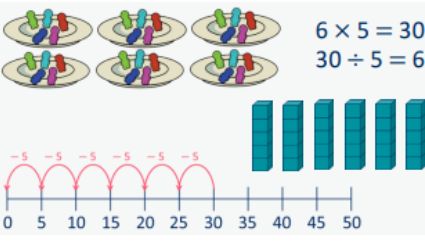
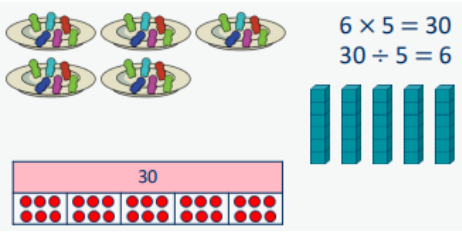
<p><b>Addition</b></p>	<ul style="list-style-type: none"> <li>Recall and use addition facts to 20 fluently, and derive and use related facts up to 100</li> <li>Add numbers using concrete objects, pictorial representations, and mentally, including:             <ul style="list-style-type: none"> <li>a two-digit number and 1s</li> <li>a two-digit number and 10s</li> <li>2 two-digit numbers</li> <li>adding 3 one-digit numbers</li> </ul> </li> <li>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>		
<p><b>Key skill</b></p>	<p><b>Key representations</b></p>		
<p><b>Add ones to any number (related facts)</b></p> <p>Make links to known facts.</p>	<p>I know that ... and ... = ... so ... and ... = ...</p> 	<p>... more than ... is ... so ... more than ... is ...</p> 	<p>What do you notice? Can you continue the pattern?</p> <div style="border: 1px solid gray; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">5 + 2 = 7</math> <math display="block">15 + 2 = 17</math> <math display="block">25 + 2 = 27...</math> </div>
<p><b>Add three 1-digit numbers</b></p> <p>Prompt children to understand that addition can be done in any order and to make links to known facts.</p>	<p>... and ... are a bond to 10 10 + ... = ...</p> 	<p>Double ... + ... = ...</p> 	<p>What do you notice? Which addition is the easiest to calculate?</p> <div style="border: 1px solid gray; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">8 + 9 + 1 =</math> <math display="block">8 + 1 + 9 =</math> <math display="block">9 + 1 + 8 =</math> </div>
<p><b>Add across a 10</b></p> <p>Partition the number being added to make a full ten.</p>	<p>... can be partitioned into ... and ...</p> 	<p>I add ... to get to ... then I add ...</p> <div style="border: 1px solid gray; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">8 + 5 = 13</math> <math display="block">28 + 5 = 33</math> </div> 	
<p><b>Add multiples of 10</b></p> <p>Make links to known facts within ten.</p>	<p>... ones + ... ones = ... ones so ... tens + ... tens = ... tens</p> <p>What is the same? What is different?</p>		

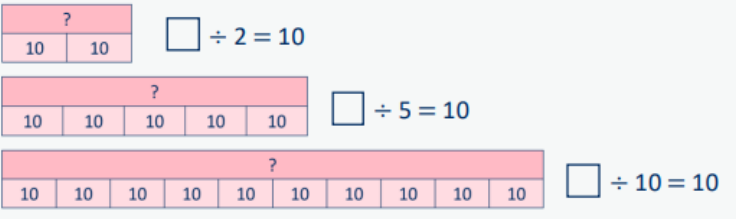
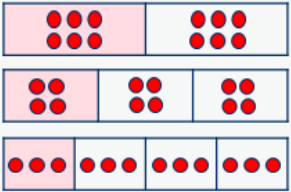

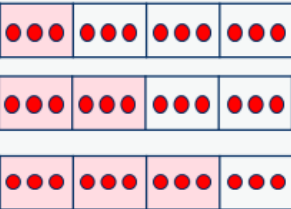
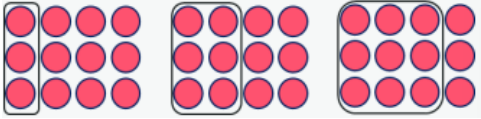
	 $3 + 2 = 5$ $30 + 20 = 50$																																																														
<p><b>Add 10s to any number</b></p> <p>Make links to known facts.</p>	<p>... tens + ... tens = ... tens ... tens and ... ones = ...</p> 	<p>To add ... I need to add 10 ... times</p> <table border="1" data-bbox="774 616 1093 806"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	<p>I know that ... and ... = ... so ... and ... = ...</p> $30 + 20 = 50$ $34 + 20 = 54$
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<p><b>Add 2-digit numbers (not across a ten)</b></p> <p>Lining up ones and tens in columns will support with later written methods.</p>	<p>... ones + ... ones = ... ones ... tens + ... tens = ... tens</p>	 <p>3 ones + 1 one = 4 ones 4 tens + 2 tens = 6 tens 6 tens + 4 ones = 64</p>																																																													
<p><b>Add 2-digit numbers (across a ten)</b></p> <p>Begin to exchange 10 ones for 1 ten</p>	<p>There are .... ones, so I do/do not need to make an exchange. ... ones = ... ten and ... ones</p> 	 <p>5 ones + 7 ones = 12 ones 12 ones = 1 ten and 2 ones 4 tens + 3 tens + 1 ten = 8 tens 8 tens and 2 ones = 82</p>																																																													
<p><b>Missing numbers</b></p> <p>Solve missing number problems and use the inverse to check.</p>	<p>How many more do you need to make ...?</p>  $6 + \square = 10$ $10 - \square = 6$	<p>If ... is a whole and ... is a part, then ... is the other part.</p>  $\square + 3 = 7$ $7 - 3 = \square$	<p>... can be partitioned into ... and ...</p> $10 + 8 = 12 + \square$ 																																																												
<p><b>Subtraction</b></p>	<ul style="list-style-type: none"> <li>Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100</li> <li>Subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> <li>a two-digit number and 1s</li> </ul> </li> </ul>																																																														

	<ul style="list-style-type: none"> <li>• a two-digit number and 10s</li> <li>• 2 two-digit numbers</li> </ul> <p>• Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems</p>																																																														
<b>Key skill</b>	<b>Key representation</b>																																																														
<b>Subtract ones from any number</b> (Related facts)  Make links to known facts.	I know that ... minus ... = ... so ... minus ... = ...  	... less than ... is ... so ... less than ... is ...  	What do you notice? Can you continue the pattern?  $8 - 3 = 5$  $18 - 3 = 15$  $28 - 3 = 25$																																																												
<b>Subtract across a 10</b>  Partition the number being subtracted to bridge through a ten.	... can be partitioned into ... and ...  	Make links with related facts.  																																																													
<b>Subtract multiples of 10</b>  Make links to known facts within ten.	... ones - ... ones = ... ones so ... tens - ... tens = ... tens  	What is the same? What is different?  	  <table border="1" data-bbox="1252 1489 1476 1624"> <tr><td colspan="2">5</td></tr> <tr><td>2</td><td>?</td></tr> <tr><td colspan="2">50</td></tr> <tr><td>20</td><td>?</td></tr> </table>	5		2	?	50		20	?																																																				
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<b>Subtract 10s from any number</b>  Make links to known facts.	... tens - ... tens = ... tens ... tens and ... ones = ...  	To subtract ... I need to subtract 10 ... times.  <table border="1" data-bbox="769 1780 1125 1993"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	I know that ... minus ... = ... so ... minus ... = ...  $50 - 20 = 30$  $54 - 20 = 34$
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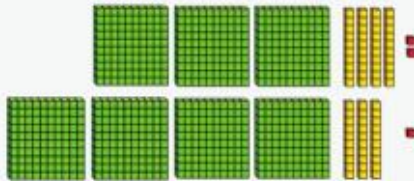

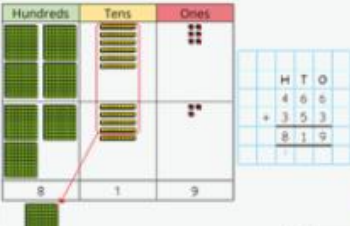

<p><b>Subtract two 2-digit numbers</b> (not across a ten)</p>	<p>... ones – ... ones = ... ones</p> <p>... tens – ... tens = ... tens</p> 	 <p>3 ones – 1 one = 2 ones</p> <p>4 tens – 2 tens = 2 tens</p> <p>2 tens and 2 ones = 22</p>	
<p><b>Subtract two 2-digit numbers</b> (across a ten)</p> <p>Begin to exchange 1 ten for 10 ones.</p>	<p>I need to make an exchange because I do not have enough ones to subtract ... ones.</p> 	 <p>3 ones – 5 ones (I need to exchange 1 ten for 10 ones)</p> <p>13 ones – 5 ones = 8 ones</p> <p>3 tens – 2 tens = 1 ten</p> <p>1 ten and 8 ones = 18</p>	
<p><b>Missing numbers</b></p> <p>Solve missing number problems and use the inverse to check.</p>	<p>How many do you need to subtract to make ...?</p>  <p><math>10 - \square = 6</math></p> <p><math>6 + \square = 10</math></p>	<p>If ... is a whole and ... is a part, then ... is the other part.</p>  <p><math>7 - 3 = \square</math></p> <p><math>\square + 3 = 7</math></p>	<p>... can be partitioned into ... and ...</p>  <p><math>18 - \square = 12 + 2</math></p>
<p><b>Multiplication</b></p>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (<math>\times</math>) and equals (=) signs.</li> <li>Show that multiplication of two numbers can be done in any order (commutative).</li> </ul>		
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Link repeated addition and multiplication</b></p> <p>Encourage children to make the link between repeated addition and multiplication.</p>	<p>There are ... equal groups with ... in each group.</p> <p>There are ... altogether.</p> 		

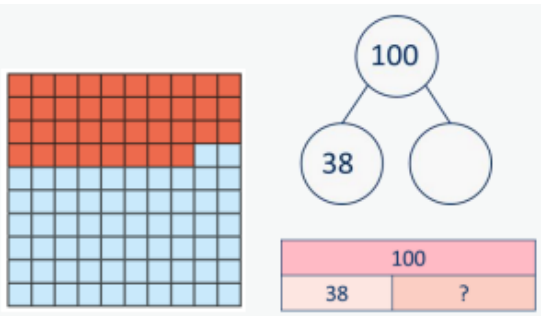
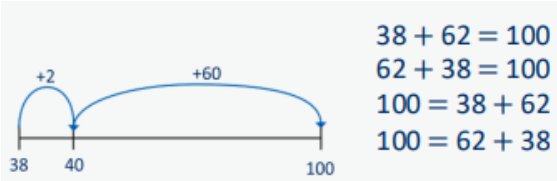
<p><b>Use arrays</b></p> <p>Encourage children to see that multiplication is commutative.</p>	<p>There are ... rows with ... in each row. There are ... columns with ... in each column.</p> 	<p>I can see ... <math>\times</math> ... and ... <math>\times</math> ...</p> <div style="border: 1px solid gray; padding: 5px; width: fit-content; margin: 10px auto;"> <math>3 \times 5 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 5 \times 3</math> </div>																																								
<p><b>Double</b></p> <p>Encourage children to make links with related facts.</p>	<p>Double ... is ...</p>  <p>Double 4 = 4 + 4 Double 4 is 8</p>	<p>Double ... is ... so double ... is ...</p>  <p>Double 4 is 8 Double 40 is 80</p>																																								
<p><b>The 2 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice that all multiples of 2 are even numbers.</p>	<p>... lots of 2 = </p> <p>... <math>\times</math> 2 =</p> 	<p>... times 2 is equal to ...</p> <table border="1" data-bbox="1046 898 1318 976"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table> <p><math>1 \times 2 = 2</math>   <math>2 = 1 \times 2</math>  <math>2 \times 2 = 4</math>   <math>4 = 2 \times 2</math>  <math>3 \times 2 = 6</math>   <math>6 = 3 \times 2</math></p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30										
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<p><b>The 10 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.</p>	<p>... lots of 10 = </p> <p>... <math>\times</math> 10 =</p> 	<p>... times 10 is equal to ...</p> <table border="1" data-bbox="1046 1301 1334 1402"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> </table> <p><math>1 \times 10 = 10</math>   <math>10 = 1 \times 10</math>  <math>2 \times 10 = 20</math>   <math>20 = 2 \times 10</math>  <math>3 \times 10 = 30</math>   <math>30 = 3 \times 10</math></p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
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21	22	23	24	25	26	27	28	29	30																																	
31	32	33	34	35	36	37	38	39	40																																	
<p><b>The 5 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.</p>	<p>... lots of 5 = </p> <p>... <math>\times</math> 5 =</p> 	<p>... times 5 is equal to ...</p> <table border="1" data-bbox="1046 1704 1318 1805"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> </table> <p><math>1 \times 5 = 5</math>   <math>5 = 1 \times 5</math>  <math>2 \times 5 = 10</math>   <math>10 = 2 \times 5</math>  <math>3 \times 5 = 15</math>   <math>15 = 3 \times 5</math></p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
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<p><b>Missing numbers</b></p> <p>Make links to known facts.</p>	<p>... is equal to ... groups of ...</p> <p>18 socks, how many pairs? </p> 	<p>... times ... is equal to ...</p> <p><math>\square \times 2 = 18</math></p> <p><math>18 = 2 \times \square</math></p>
<p><b>Division</b></p> <ul style="list-style-type: none"> <li>Recall and use division facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for division within the multiplication tables and write them using the division (<math>\div</math>) and equals (=) signs.</li> <li>Recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a quantity.</li> </ul>		
<p><b>Key skills</b></p> <p><b>Key representations</b></p>		
<p><b>Divide by 2</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts and halving.</p>	<p>There are ... equal groups of 2</p> <p><math>\dots \div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>	<p>... shared equally between 2 is ...</p> <p>Half of ... is ...</p> <p><math>\dots \div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>
<p><b>Divide by 10</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 10</p> <p><math>\dots \div 10 = \dots</math></p>  <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p>	<p>... shared equally between 10 is ...</p> <p><math>\dots \div 10 = \dots</math></p>  <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p>
<p><b>Divide by 5</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 5</p> <p><math>\dots \div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>	<p>... shared equally between 5 is ...</p> <p><math>\dots \div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>
<p><b>Missing numbers</b></p> <p>Bar models are useful to show the link</p>	<p>... divided by 2/ /10 is equal to ...</p>	


<p>between multiplication and division.</p>		
<p><b>Unit fractions</b></p> <p>In Y2 the focus is on finding <math>\frac{1}{2}</math>, <math>\frac{1}{4}</math> and <math>\frac{1}{3}</math></p> <p>Bar models are useful to show the link between division and finding a fraction.</p>	<p>The objects have been shared fairly into ... groups.</p> <p><math>\frac{1}{\square}</math> of ... is ...</p> 	<p>There are ... equal parts. There is ... part circled.</p> <p><math>\frac{1}{\square}</math> is circled.</p> 
<p><b>Non-unit fractions</b></p> <p>In Y2 the focus is on finding <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math></p> <p>Prompt children to notice that <math>\frac{2}{4}</math> is equivalent to <math>\frac{1}{2}</math></p>	<p>The objects have been shared fairly into ... groups.</p> <p><math>\frac{\square}{\square}</math> of ... is ...</p> 	<p>There are ... equal parts. There are ... parts circled.</p> <p><math>\frac{\square}{\square}</math> is circled.</p> 

## Year 3

<p><b>Addition</b></p>	<ul style="list-style-type: none"> <li>• Add numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.</li> <li>• Add numbers with up to three digits, using formal written methods of columnar addition.</li> <li>• Add fractions with the same denominator within 1 whole.</li> <li>• Calculate the time taken by particular events or tasks.</li> </ul>																															
<p><b>Key skill</b></p>	<p><b>Key representations</b></p>																															
<p><b>Add 1s, 10s or 100s to a 3-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds column will increase by ...</p> <div style="display: flex; justify-content: space-around;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr> </thead> <tbody> <tr><td>■ ■ ■</td><td>■ ■ ■ ■</td><td>■ ■</td></tr> <tr><td>■ ■ ■</td><td></td><td></td></tr> </tbody> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td>100 100</td><td>10 10</td><td>1 1</td></tr> <tr><td>100 100</td><td>10 10</td><td>1 1</td></tr> <tr><td>100 100</td><td>10 10</td><td>1 1</td></tr> <tr><td>100</td><td>10</td><td>1</td></tr> </tbody> </table> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: left;"> <math>444 + 5 =</math>  <math>444 + 50 =</math>  <math>444 + 500 =</math> </div> <div style="text-align: left;"> <math>777 + 2 =</math>  <math>777 + 20 =</math>  <math>777 + 200 =</math> </div> </div>	Hundreds	Tens	Ones	■ ■ ■	■ ■ ■ ■	■ ■	■ ■ ■			H	T	O	100 100	10 10	1 1	100 100	10 10	1 1	100 100	10 10	1 1	100	10	1	<p>What patterns do you notice?</p> <div style="margin-top: 10px;"> <math>235 + 3 =</math>  <math>235 + 30 =</math>  <math>235 + 300 =</math> </div> <div style="margin-top: 10px;"> <math>111 + \square = 118</math>  <math>604 + 20 = 111 + \square = 181</math>  <math>604 + 50 = 111 + \square = 811</math>  <math>604 + 90 = 111 + \square = 811</math> </div>						
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<p><b>Add two numbers (no exchange)</b></p> <p>Mental strategies and introduction of formal written method.</p>	<p>... ones + ... ones = ... ones          ... tens + ... tens = ... tens          ... hundreds + ... hundreds = ... hundreds</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr><th colspan="2">?</th></tr> <tr><td>345</td><td>432</td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td>3</td><td>4</td><td>5</td></tr> <tr><td>+</td><td>4</td><td>3</td><td>2</td></tr> <tr><td colspan="4">-----</td></tr> </tbody> </table> </div>		?		345	432	H	T	O	3	4	5	+	4	3	2	-----															
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<p><b>Add two numbers across a 10 or 100</b></p> <p>Formal written method involving up to 2 exchanges including 3-digit plus 2-digit numbers.</p>	<p>There are ... ones, so I do/do not need to make an exchange.          There are ... tens, so I do/do not need to make an exchange.          ... ones = ... ten and ... ones.          ... tens = ... hundred and ... tens.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr><th colspan="2">?</th></tr> <tr><td>255</td><td>54</td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td>2</td><td>5</td><td>5</td></tr> <tr><td>+</td><td>5</td><td>4</td></tr> <tr><td colspan="3">-----</td></tr> </tbody> </table> </div> <div style="margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td>3</td><td>6</td><td>7</td></tr> <tr><td>+</td><td>1</td><td>6</td><td>4</td></tr> <tr><td colspan="4">-----</td></tr> </tbody> </table> </div>		?		255	54	H	T	O	2	5	5	+	5	4	-----			H	T	O	3	6	7	+	1	6	4	-----			
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<p><b>Complements to 100</b></p> <p>Pairs of numbers which total 100</p>	<p>... plus ... is equal to 100</p> 	<p>I add ... to get to the next 10, then ... to get to 100</p> 
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<p><b>Add fractions with the same denominator within 1 whole</b></p> <p>Make links with known facts.</p>	<p>When adding fractions with the same denominator, I only add the numerator.</p> <p>... fifths + ... fifths = ... fifths</p> 
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<p><b>Calculate the duration of events</b></p> <p>Find durations of time between a given start and end point.</p> <p>Children will need to calculate complements to 60</p>	<p>From ... to ... o'clock is ... minutes.</p> <p>From ... o'clock to ... is ... minutes.</p> <p>The total time taken is ... minutes.</p> 
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<p><b>Subtraction</b></p>	<ul style="list-style-type: none"> <li>Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.</li> <li>Subtract numbers with up to three digits, using formal written methods.</li> <li>Subtract fractions with the same denominator within 1 whole.</li> </ul>
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<p><b>Key skills</b></p>	<p><b>Key representations</b></p>
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<p><b>Subtract 1s, 10s and 100s from a 3-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt</p>	<p>The ones/tens/hundreds column will decrease by ...</p> 	<p>What patterns do you notice?</p> <p>235 - 3 =</p> <p>235 - 30 =</p> <p>235 - 300 =</p>
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children to notice which digit changes.	$444 - 2 =$	$777 - 4 =$	$118 - \square = 111$ $181 - \square = 111$ $811 - \square = 111$
	$444 - 20 =$	$777 - 40 =$	
	$444 - 200 =$	$777 - 400 =$	

**Subtract two numbers (no exchange)**

... ones - ... ones = ... ones  
 ... tens - ... tens = ... tens  
 ... hundreds - ... hundreds = ... hundreds

Mental strategies and introduction of formal written method.

**Subtract two numbers across a 10 or 100**

I need to subtract ... ones. I do/do not need to make an exchange.  
 I need to subtract ... tens. I do/do not need to make an exchange.  
 I can exchange 1 ... for 10 ...

Formal written method involving up to 2 exchanges including 3-digit subtract 2-digit numbers

**Complements to 100**

Focus on subtraction facts. Encourage children to notice patterns.

100 minus ... is equal to ...

I subtract ... tens, then I subtract ... ones.

$100 - 38 = 62$   
 $100 - 62 = 38$   
 $62 = 100 - 38$   
 $38 = 100 - 62$

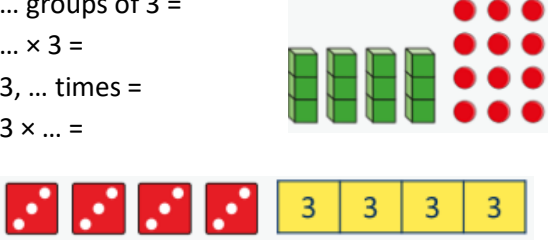
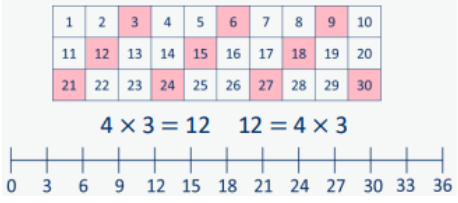
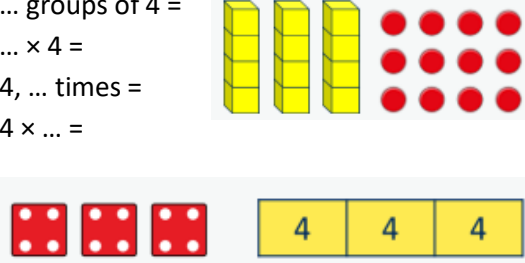
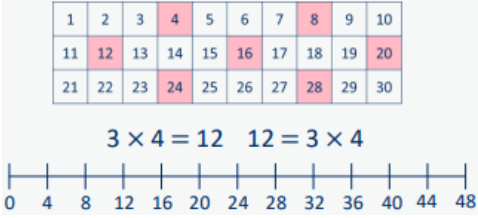
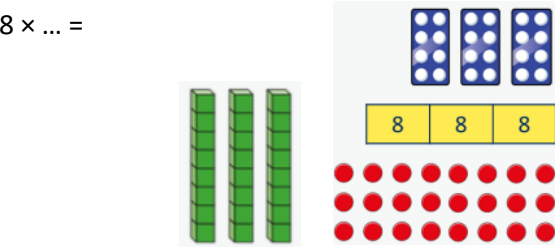
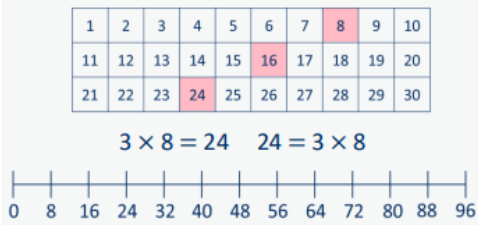

**Subtract fractions with the same denominator within 1 whole**

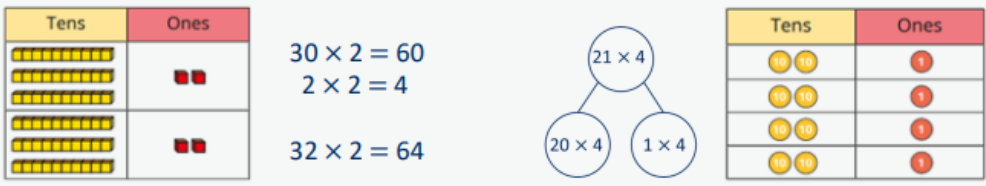
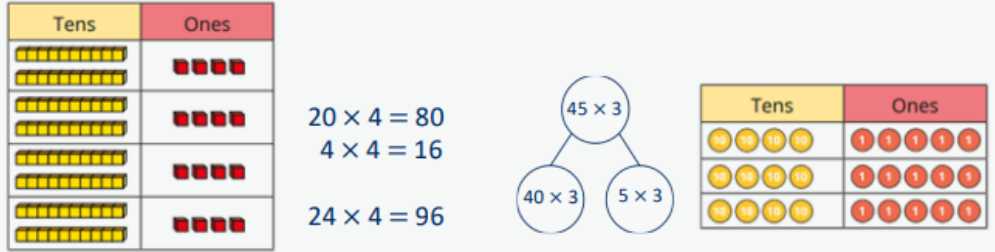

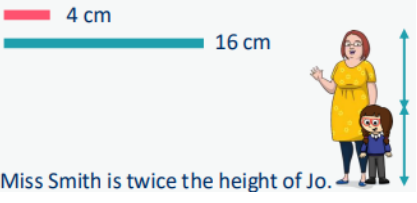
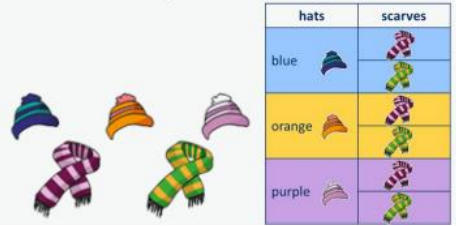
When subtracting fractions with the same denominator, I only subtract the numerator. ...

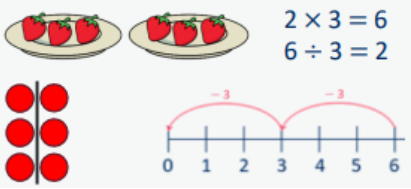
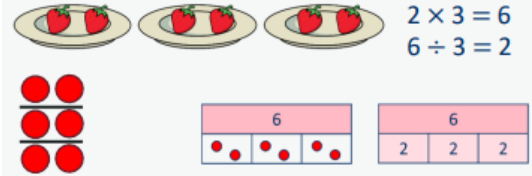
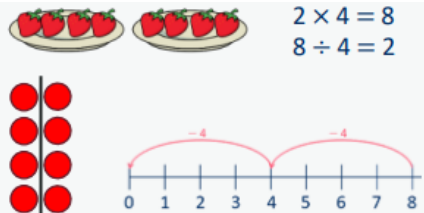
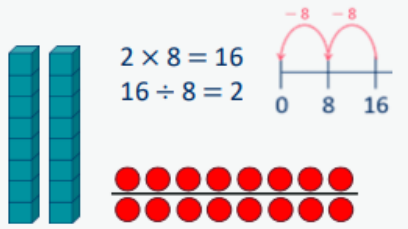
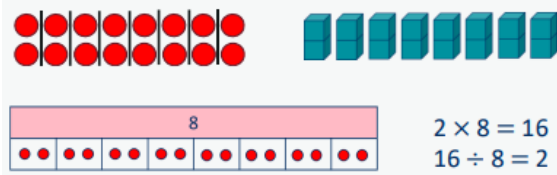
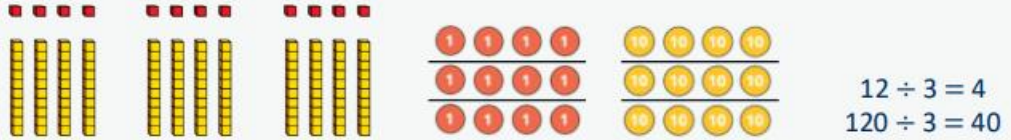
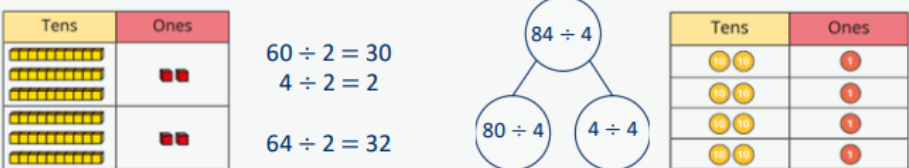
fifths - ... fifths = ... fifths

$\frac{5}{5} - \frac{1}{5}$   
 $\frac{4}{5} - \frac{1}{5}$   
 $\frac{3}{5} - \frac{1}{5}$

$\frac{7}{7}$   
 $\frac{2}{7}$

<b>Multiplication</b>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.</li> </ul>	
<b>Key skills</b>	<b>Key representations</b>	
<b>The 3 times-table</b>  Encourage daily counting in multiples both forwards and back.	<p>... groups of 3 =  ... <math>\times</math> 3 =  3, ... times =  3 <math>\times</math> ... =</p> 	<p>... times 3 is equal to ...</p> 
<b>The 4 times-table</b>  Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2 and 4 times-tables.	<p>... groups of 4 =  ... <math>\times</math> 4 =  4, ... times =  4 <math>\times</math> ... =</p> 	<p>... times 4 is equal to ...</p> 
<b>The 8 times-table</b>  Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2, 4 and 8 times-tables.	<p>... lots of 8 =  ... <math>\times</math> 8 =  8, ... times =  8 <math>\times</math> ... =</p> 	<p>... times 8 is equal to ...</p> 
<b>Related facts</b>  Use knowledge of multiplying by 10 to scale times-table facts.	<p>... <math>\times</math> ... ones is equal to ... ones  so ... <math>\times</math> ... tens is equal to ... tens.</p> 	
<b>Multiply a 2-digit number by a 1-digit number - no exchange</b>	<p>... tens multiplied by ... is equal to ... tens.  ...ones multiplied by ... is equal to ... ones.</p>	

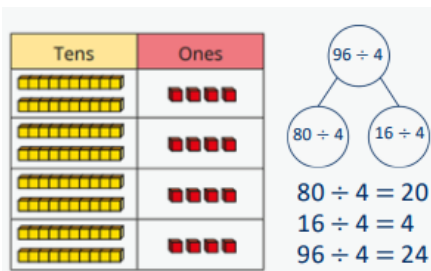
<p>Children apply their understanding of partitioning to represent and solve calculations using the expanded method.</p>	 <p> <math>30 \times 2 = 60</math>  <math>2 \times 2 = 4</math>  <math>32 \times 2 = 64</math> </p> <p> <math>21 \times 4</math>  <math>20 \times 4</math>   <math>1 \times 4</math> </p>	
<p><b>Multiply a 2-digit number by a 1-digit number - with exchange</b></p> <p>Children apply their understanding of partitioning to represent and solve calculations using the expanded method.</p>	<p>... tens multiplied by ... is equal to ... tens. ... ones multiplied by ... is equal to ... ones.</p>  <p> <math>20 \times 4 = 80</math>  <math>4 \times 4 = 16</math>  <math>24 \times 4 = 96</math> </p> <p> <math>45 \times 3</math>  <math>40 \times 3</math>   <math>5 \times 3</math> </p>	
<p><b>Scaling</b></p> <p>Children focus on multiplication as scaling ( ... times the size) as opposed to repeated addition.</p>	<p>There are ... times as many ... as ...</p>  <p>There are 3 times as many triangles as circles.</p>	<p>... is ... times the size of ... ... is ... times the length/height of ...</p>  <p>Miss Smith is twice the height of Jo.</p>
<p><b>Correspondence problems</b> (How many ways?)</p> <p>Encourage children to work systematically to find all the different possible combinations.</p>	<p>For every ... , there are ... possible ... There are ... <math>\times</math> ... possibilities altogether.</p>  <p>For every hat, there are two possible scarves. <math>3 \times 2 = 6</math></p> <p>There are 6 possibilities altogether.</p>	
<p><b>Division</b></p>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.</li> </ul>	
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>	

<p><b>Divide by 3</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... groups of 3 in ...</p> <p>... <math>\div</math> 3 =</p> 	<p>... has been shared equally into 3 equal groups.</p> <p>... <math>\div</math> 3 =</p> 
<p><b>Divide by 4</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... groups of 4 in ...</p> <p>... <math>\div</math> 4 =</p> 	<p>... has been shared equally into 4 equal groups.</p> <p>... <math>\div</math> 4 =</p> 
<p><b>Divide by 8</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... groups of 8 in ...</p> <p>... <math>\div</math> 8 =</p> 	<p>... has been shared equally into 8 equal groups.</p> <p>... <math>\div</math> 8 =</p> 
<p><b>Related facts</b></p> <p>Link to known times-table facts.</p>	<p>... <math>\div</math> ... is equal to ..., so ... tens <math>\div</math> ... is equal to ... tens.</p> 	
<p><b>Divide a 2-digit number by a 1-digit number - no exchange</b></p> <p>Partition into tens and ones to divide and then recombine.</p>	<p>... tens divided by ... is equal to ... tens. ... ones divided by ... is equal to ... ones.</p> 	

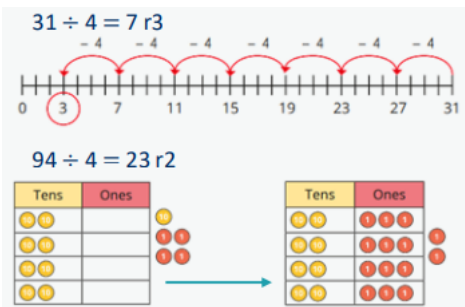
**Divide a 2-digit number by a 1-digit number - with remainders**

Encourage children to partition numbers flexibly to help them to divide more efficiently.

... tens divided by ... is equal to ... tens.  
 ... ones divided by ... is equal to ... ones.



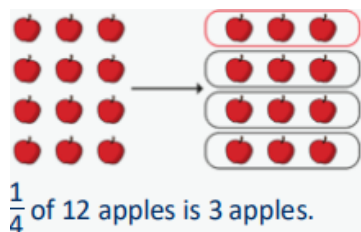
There are ... groups of ...  
 There are ... remaining.



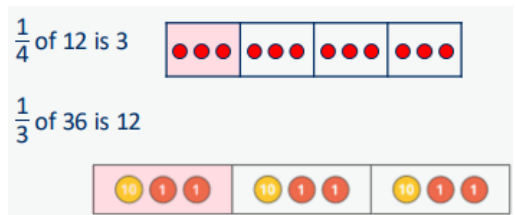
**Unit fractions of a set of objects**

Bar models are useful to show the link between division and fractions, for example, dividing by 3 and finding a third.

The whole is divided into ... equal parts.  
 Each part is  $\frac{1}{\square}$  of the whole.



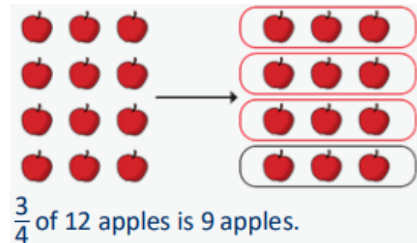
One ... of ... is ...



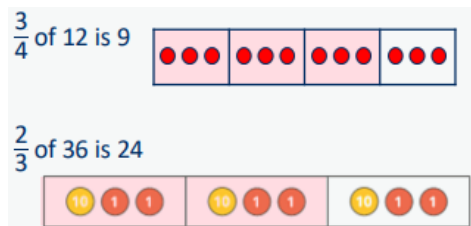
**Non-unit fractions of a set of objects**

Bar models are a useful representation and show the links with division and multiplication.

The whole is divided into ... equal parts.  
 Each part is  $\frac{1}{\square}$  of the whole.



$\frac{1}{\square}$  of ... is ..., so  $\frac{\square}{\square}$  of ... is ...



**Year 4**

**Addition**

- Add numbers with up to 4 digits using a formal written method.
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places.
- Add fractions with the same denominator.

**Key skill**      **Key representations**

**Add 1s, 10s or 100s to a 4-digit number**

Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds/thousands column will increase by ...

Thousands	Hundreds	Tens	Ones
1,000 1,000 1,000	100 100 100 100	10 10	1 1 1 1 1

$3,425 + 3 =$                        $3,425 + 300 =$   
 $3,425 + 30 =$                      $3,425 + 3,000 =$

What patterns do you notice?

$2,350 + 3 =$   
 $2,350 + 30 =$   
 $2,350 + 300 =$   
 $2,350 + 3,000 =$

$6,040 + 200 =$        $2,211 + \square = 2,251$   
 $6,040 + 500 =$        $2,211 + \square = 2,215$   
 $6,040 + 900 =$        $2,211 + \square = 2,511$

**Add up to two 4-digit numbers**

Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.

There are ... ones/tens/hundreds so I do/do not need to make an exchange. I can exchange 10 ... for 1 ...


Th	H	T	O
1000 1000 1000	100 100 100 100 100 100	10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

Th	H	T	O
4	6	7	3
+ 1 5 1 8			
<hr/>			
6	1	9	1
1	1		

**Add decimal numbers in the context of money**


Emphasis on partitioning and use of number lines rather than formal written calculations.

... pence + ... pence = ... pence  
... pounds + ... pounds = ... pounds



$45p + 25p = 70p$   
 $£2 + £3 = £5$   
 $£5 + 70p = £5.70$

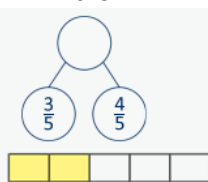
£3.25 can be partitioned into £3 + 20p + 5p

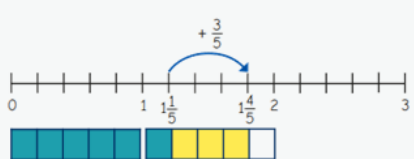


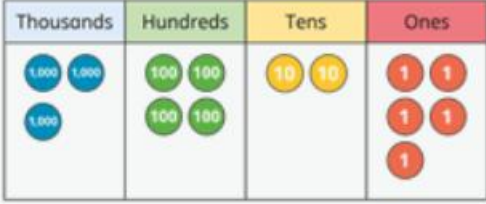
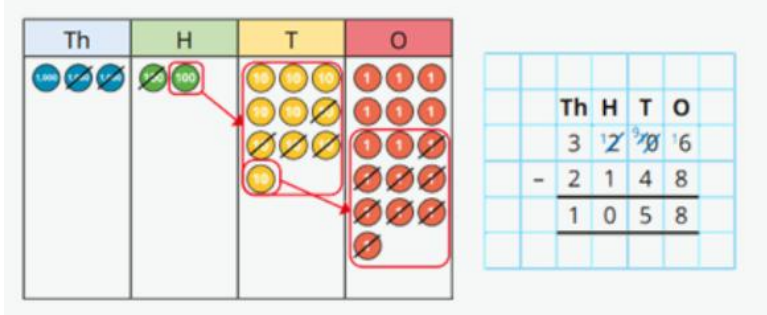
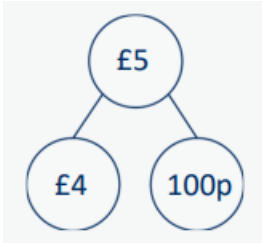
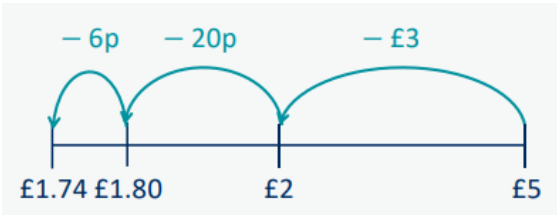
**Add fractions and mixed numbers with the same denominator beyond 1 whole**

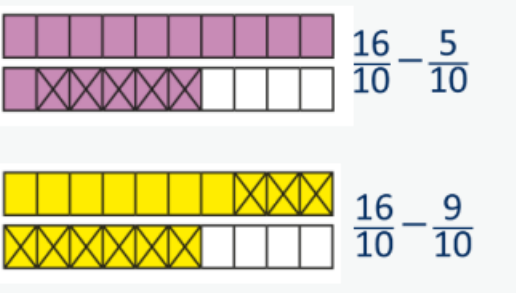
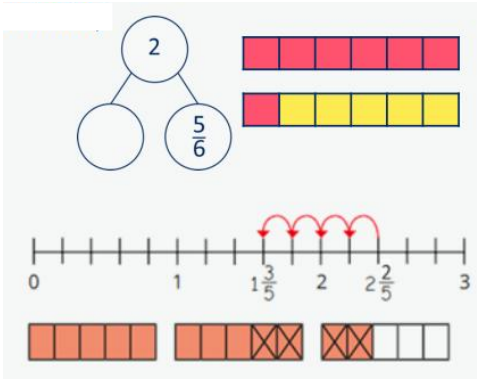
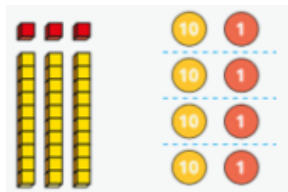
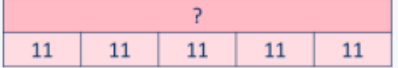
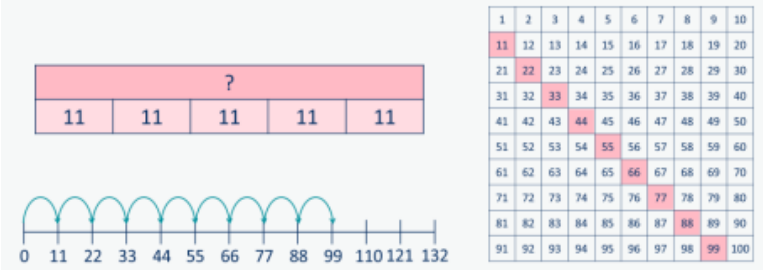
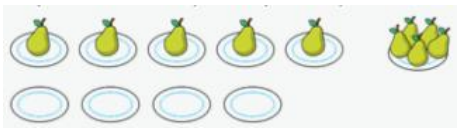
When adding fractions with the same denominator, I only add the numerator.  
... fifths + ... fifths = ... fifths


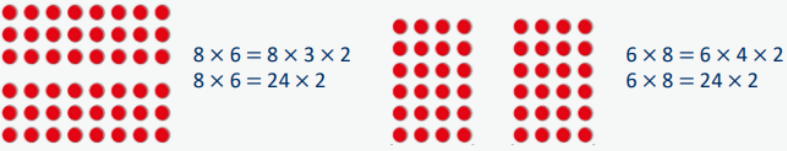
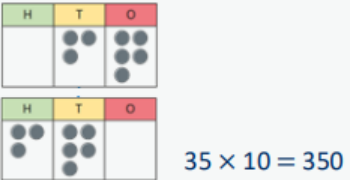
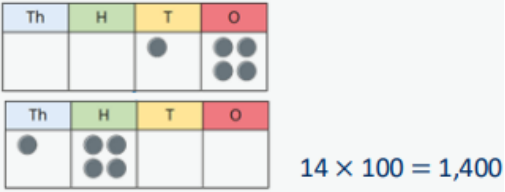
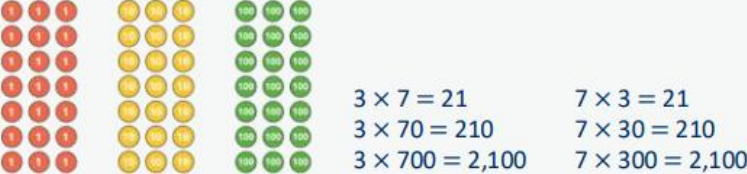
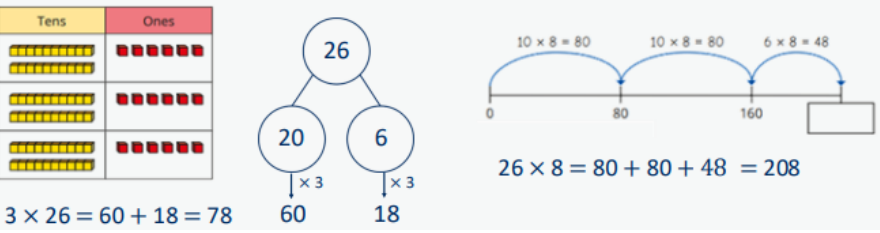
$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$




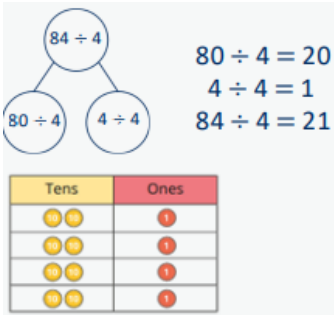
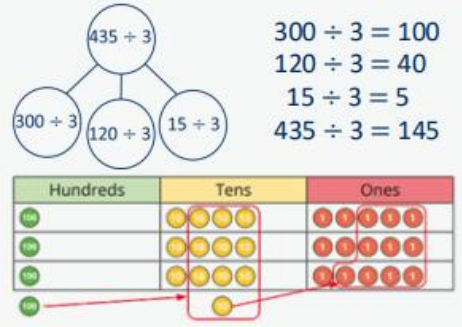
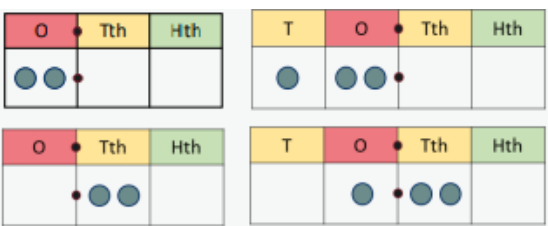
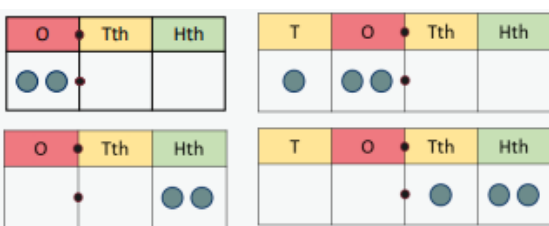


<p><b>Subtraction</b></p>	<ul style="list-style-type: none"> <li>• Subtract numbers with up to 4 digits using a formal written method.</li> <li>• Solve simple measure and money problems involving fractions and decimals to 2 decimal places.</li> <li>• Subtract fractions with the same denominator.</li> </ul>	
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Subtract 1s, 10s, 100s and 1,000s from a 4-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds/thousands column will decrease by ...</p>  <p>3,425 - 2 =                      3,425 - 200 =</p> <p>3,425 - 20 =                    3,425 - 2,000 =</p>	<p>What patterns do you notice?</p> <p>4,356 - 3 =</p> <p>4,356 - 30 =</p> <p>4,356 - 300 =</p> <p>4,356 - 3,000 =</p> <p>6,940 - 200 =</p> <p>6,940 - 300 =</p> <p>6,940 - 400 =</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>4,433 - <input type="text"/> = 4,430</p> <p>4,433 - <input type="text"/> = 4,033</p> <p>4,433 - <input type="text"/> = 4,403</p> </div>
<p><b>Subtract up to two 4-digit numbers</b></p> <p>Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.</p>	<p>I need to subtract... ones/tens/hundreds. I do/do not need to make an exchange. I can exchange 1... for 10...</p> 	
<p><b>Subtract decimal numbers in the context of money</b></p> <p>Emphasis here is on partitioning and use of number lines rather than formal written calculations.</p>	<p>I can partition £... into £... and 100p</p> <p>£... - £... = £...</p> <p>100p - ...p = ...p</p> <p>£5 - £3.26</p> <p>£4 - £3 = £1</p> <p>100p - 26p = 74p</p> <p>£5 - £3.26 = £1.74</p> 	<p>£3.26 can be partitioned into £3 + 20p + 6p</p> 

<p><b>Subtract fractions and mixed numbers with the same denominator</b></p> <p>Include subtracting fractions from wholes.</p>	<p>When subtracting fractions with the same denominator, I only subtract the numerator. ... tenths - ... tenths = ... tenths</p>  								
<p><b>Multiplication</b></p>	<ul style="list-style-type: none"> <li>Recall multiplication facts for multiplication tables up to <math>12 \times 12</math></li> <li>Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers.</li> <li>Recognise and use factor pairs and commutativity in mental calculations.</li> <li>Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.</li> <li>Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as <math>n</math> objects are connected to <math>m</math> objects.</li> </ul>								
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>								
<p><b>Times-table facts to <math>12 \times 12</math></b></p> <p>Encourage daily counting in multiples both forwards and back. Encourage children to notice links between related times-tables.</p>	<p>... groups of ... = ... times ... is equal to ... ... <math>\times</math> ... =</p>   								
<p><b>Multiply by 1 and 0</b></p>	<p>Any number multiplied by 1 is equal to ... Any number multiplied by 0 is equal to ...</p>  <p>... <math>\times</math> ... = ...</p> <table border="1" data-bbox="1109 1568 1404 1713"> <tbody> <tr> <td><math>1 \times 1 = 1</math></td> <td><math>1 \times 0 = 0</math></td> </tr> <tr> <td><math>2 \times 1 = 2</math></td> <td><math>2 \times 0 = 0</math></td> </tr> <tr> <td><math>3 \times 1 = 3</math></td> <td><math>3 \times 0 = 0</math></td> </tr> <tr> <td><math>4 \times 1 = 4</math></td> <td><math>4 \times 0 = 0</math></td> </tr> </tbody> </table>	$1 \times 1 = 1$	$1 \times 0 = 0$	$2 \times 1 = 2$	$2 \times 0 = 0$	$3 \times 1 = 3$	$3 \times 0 = 0$	$4 \times 1 = 4$	$4 \times 0 = 0$
$1 \times 1 = 1$	$1 \times 0 = 0$								
$2 \times 1 = 2$	$2 \times 0 = 0$								
$3 \times 1 = 3$	$3 \times 0 = 0$								
$4 \times 1 = 4$	$4 \times 0 = 0$								
<p><b>Multiply 3 numbers</b></p> <p>Children use their understanding of commutativity to</p>	<p>To work out ... <math>\times</math> ... <math>\times</math> ..., I can first calculate ... <math>\times</math> ... and then multiply the answer by ...</p>								

<p>multiply more efficiently.</p>	 $4 \times 2 \times 3 = 8 \times 3 = 24$ $2 \times 3 \times 4 = 6 \times 4 = 24$ $3 \times 4 \times 2 = 12 \times 2 = 24$	
<p><b>Factor pairs</b></p> <p>Children explore equivalent calculations using different factors pairs.</p>	<p><math>12 = \dots \times \dots</math>, so <math>\dots \times 12 = \dots \times \dots \times \dots</math></p>  $8 \times 6 = 8 \times 3 \times 2$ $8 \times 6 = 24 \times 2$ $6 \times 8 = 6 \times 4 \times 2$ $6 \times 8 = 24 \times 2$	
<p><b>Multiply by 10 and 100</b></p> <p>Some children may overgeneralise that multiplying by 10 or 100 always results in adding zeros. This will cause issues later when multiplying decimals.</p>	<p>When I multiply by 10, the digits move ... place value column to the left.</p> <p>... is 10 times the size of ...</p>  $35 \times 10 = 350$	<p>When I multiply by 100, the digits move ... place value columns to the left.</p> <p>... is 100 times the size of ...</p>  $14 \times 100 = 1,400$
<p><b>Related facts</b></p> <p>Use knowledge of multiplying by 10 and 100 to scale times-table facts.</p>	<p>... <math>\times</math> ... ones is equal to ... ones  so ... <math>\times</math> ... tens is equal to ... tens  and ... <math>\times</math> ... hundreds is equal to ... hundreds.</p>  $3 \times 7 = 21$ $3 \times 70 = 210$ $3 \times 700 = 2,100$ $7 \times 3 = 21$ $7 \times 30 = 210$ $7 \times 300 = 2,100$	
<p><b>Mental strategies</b></p> <p>Partition 2 or 3-digit numbers to multiply using informal methods.</p>	<p>... tens multiplied by ... is equal to ... tens.  ...ones multiplied by ... is equal to ... ones.</p>  $3 \times 26 = 60 + 18 = 78$ $26 \times 8 = 80 + 80 + 48 = 208$	
<p><b>Multiply a 2 or 3-digit number by a 1-digit number</b></p>	<p>To multiply a 2-digit number by ... , I multiply the ones by ... and the tens by ...  To multiply a 3-digit number by ... , I multiply the ones by ... , the tens by ... and the hundreds by ...</p>	

<p>The short multiplication method is introduced for the first time, initially in an expanded form.</p>																										
<p><b>Scaling</b></p> <p>Children focus on multiplication as scaling ( ... times the size).</p>	<p>... is ... times the size of ...</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>A computer mouse costs £7 A keyboard costs 6 times as much.</p> </div> <div style="text-align: center;"> <p>A red ribbon is 6 cm. A yellow ribbon is 7 times as long.</p> </div> </div>																									
<p><b>Correspondence problems</b></p> <p>Encourage children to use tables to show all the different possible combinations.</p>	<p>For every ... , there are ... possibilities. There are ... × ... possibilities altogether.</p> <p>A pizza company offers a choice of 5 toppings and 3 bases.</p> $5 \times 3 = 15$	<table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th></th> <th>Deep pan</th> <th>Italian</th> <th>Thin</th> </tr> </thead> <tbody> <tr> <th>Cheese</th> <td>C DP</td> <td>C I</td> <td>C Th</td> </tr> <tr> <th>Mushroom</th> <td>M DP</td> <td>M I</td> <td>M Th</td> </tr> <tr> <th>Vegetable</th> <td>V DP</td> <td>V I</td> <td>V Th</td> </tr> <tr> <th>Chicken</th> <td>C DP</td> <td>C I</td> <td>C Th</td> </tr> <tr> <th>Tuna</th> <td>T DP</td> <td>T I</td> <td>T Th</td> </tr> </tbody> </table>		Deep pan	Italian	Thin	Cheese	C DP	C I	C Th	Mushroom	M DP	M I	M Th	Vegetable	V DP	V I	V Th	Chicken	C DP	C I	C Th	Tuna	T DP	T I	T Th
	Deep pan	Italian	Thin																							
Cheese	C DP	C I	C Th																							
Mushroom	M DP	M I	M Th																							
Vegetable	V DP	V I	V Th																							
Chicken	C DP	C I	C Th																							
Tuna	T DP	T I	T Th																							
<p><b>Division</b></p>	<ul style="list-style-type: none"> <li>Recall division facts for multiplication tables up to <math>12 \times 12</math></li> <li>Use place value, known and derived facts to divide mentally, including: dividing by 1</li> <li>Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.</li> </ul>																									
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>																									
<p><b>Division facts to <math>12 \times 12</math></b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... groups of ... in ... <math>\dots \div \dots =</math></p>	<p>... has been shared equally into ... equal groups. <math>\dots \div \dots =</math></p>																								
<p><b>Divide a number by 1 and itself</b></p> <p>Children may try to divide a number by zero and it should be highlighted that this is not possible.</p>	<p>When I divide a number by 1, the number remains the same.</p> <p>5 shared between 1 is 5 </p> <p>There are 5 groups of 1 in 5</p>	<p>When I divide a number by itself, the answer is 1</p> <p>5 shared between 5 is 1</p> <p>There is 1 group of 5 in 5 </p>																								

<p><b>Related facts</b></p> <p>Link to known times-table facts.</p>	<p>... ÷ ... is equal to ...  so ... tens ÷ ... is equal to ... tens  and ... hundreds ÷ ... is equal to ... hundreds.</p>  <div style="display: flex; justify-content: space-around;"> <div style="text-align: left;"> <math>21 \div 7 = 3</math>  <math>210 \div 7 = 30</math>  <math>2,100 \div 7 = 300</math> </div> <div style="text-align: left;"> <math>21 \div 3 = 7</math>  <math>210 \div 3 = 70</math>  <math>2,100 \div 3 = 700</math> </div> </div>	
<p><b>Divide a 2 or 3-digit number by a 1-digit number</b></p> <p>Progress from divisions with no exchange, to divisions with exchange and then divisions with remainders.</p>	<p>I can partition ... into ... tens and ... ones.</p> 	<p>I cannot share the hundreds/tens equally, so I need to exchange 1 ... for 10 ...</p> 
<p><b>Divide by 10 and 100</b></p> <p>Encourage children to notice that dividing by 100 is the same as dividing by 10 twice.</p>	<p>When I divide by 10, the digits move 1 place value column to the right.  ... is one-tenth the size of ...</p>  <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <math>2 \div 10 = 0.2</math> </div> <div style="text-align: center;"> <math>12 \div 10 = 1.2</math> </div> </div> <p>When I divide by 100, the digits move 2 place value columns to the right.  ... is one-hundredth the size of ...</p>  <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <math>2 \div 100 = 0.02</math> </div> <div style="text-align: center;"> <math>12 \div 100 = 0.12</math> </div> </div>	

**Year 5**

- Addition**
- Add whole numbers with more than 4 digits, including using formal written methods.
  - Add numbers mentally with increasingly large numbers.
  - Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1
  - Add fractions with the same denominator, and denominators that are multiples of the same number

**Key skill**      **Key representations**

**Add using mental strategies**

Add 1s, 10s, 100s, etc. to any number.

Use number bonds and related facts.

TTh	Th	H	T	O
●●●●	●●●●●●	●●●●●●	●●●●	

$48,650 + 300 =$   
 $48,650 + 30,000 =$   
 $48,650 + 30 =$

To add ..., I can add ... then subtract ...

?
6,458    99

6,458      6,557    6,558

**Add whole numbers with more than 4 digits**

Encourage children to estimate and use inverse operations to check answers to calculations.

I can exchange 10 ... for 1 ...

TTh	Th	H	T	O
●●●●	●●●●●●	●●●●●●	●●●●	●●

2	6	5	7	4	
+	1	6	2	3	1
-----					
4	2	8	0	5	
1		1			

	4	1		
+	2	8	4	
-----				
8	9	9	2	6

**Add decimals with up to 2 decimal places**

Progress from the same number of decimal places to a different number of decimal places, and from no exchange to exchange.

I do/do not need to make an exchange because ...

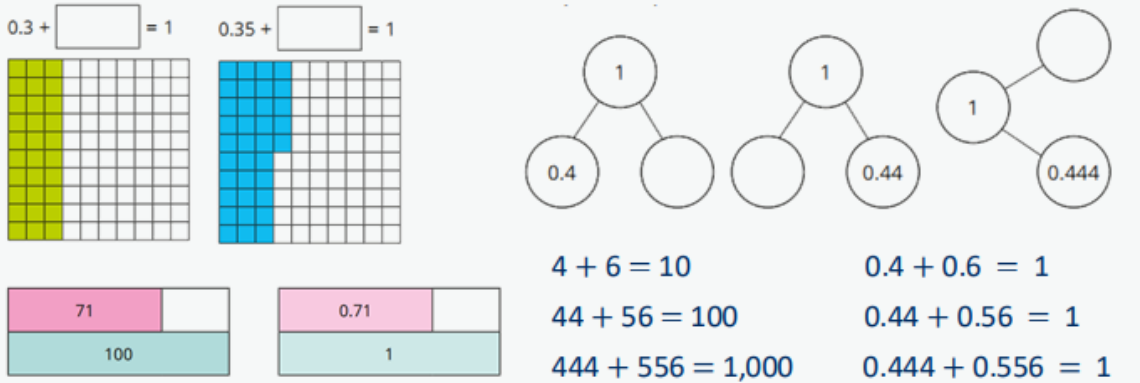
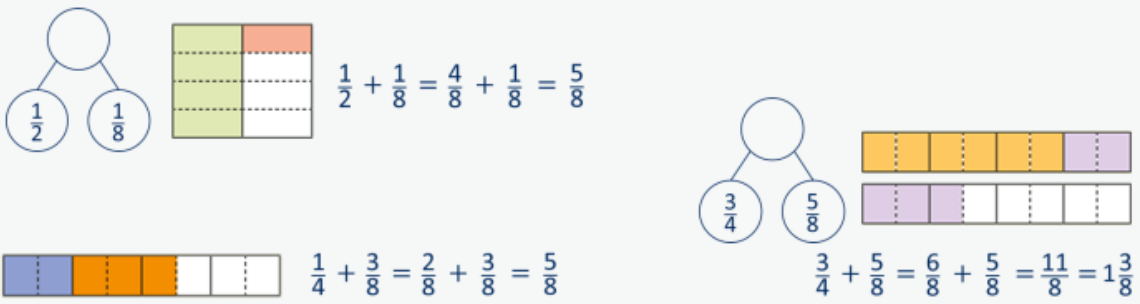
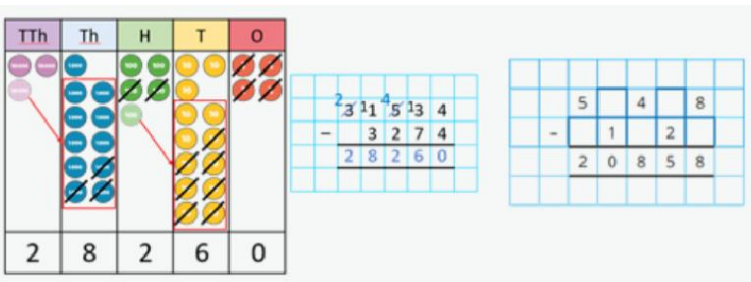
I can exchange 10 ... for 1 ...

Ones	Tenths	Hundredths
●●●	●●●●	●●●●●●

4	4	5	
+	3	2	1
-----			

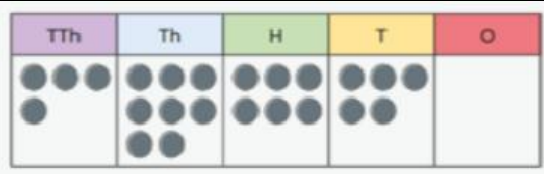
O	Tth	Hth	Thth
●	●●	●●●●	●●●●

1	2	8	1
+	2	5	4
-----			

<p><b>Complements to 1</b></p> <p>Pairs of numbers with up to 3 decimal places which total 1</p> <p>Encourage children to make links with bonds to 10 and complements to 100 and 1,000</p>	 <p> <math>0.3 + \square = 1</math>     <math>0.35 + \square = 1</math> </p> <p> <math>4 + 6 = 10</math>     <math>0.4 + 0.6 = 1</math>  <math>44 + 56 = 100</math>     <math>0.44 + 0.56 = 1</math>  <math>444 + 556 = 1,000</math>     <math>0.444 + 0.556 = 1</math> </p>										
<p><b>Add fractions with denominators that are a multiple of one another</b></p> <p>Encourage children to convert fractions to the same denominator before adding.</p> <p>Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.</p>	<p>The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.</p>  <p> <math>\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}</math> </p> <p> <math>\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}</math> </p> <p> <math>\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}</math> </p>										
<p><b>Subtraction</b></p>	<ul style="list-style-type: none"> <li>• Subtract whole numbers with more than 4 digits.</li> <li>• Subtract numbers mentally with increasingly large numbers.</li> <li>• Subtract decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1</li> <li>• Subtract fractions with the same denominator, and denominators that are multiples of the same number.</li> </ul>										
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>										
<p><b>Subtract whole numbers with more than 4 digits</b></p> <p>Encourage children to estimate and use inverse operations to check answers to calculations</p>	<p>I can exchange 1 ... for 10 ...</p>  <p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td>2</td><td>8</td><td>2</td><td>6</td><td>0</td></tr> </table>   <math display="block">\begin{array}{r} 28260 \\ - 315134 \\ \hline 28260 \end{array}</math>   <math display="block">\begin{array}{r} 548 \\ - 12 \\ \hline 536 \end{array}</math> </p>	TTh	Th	H	T	O	2	8	2	6	0
TTh	Th	H	T	O							
2	8	2	6	0							

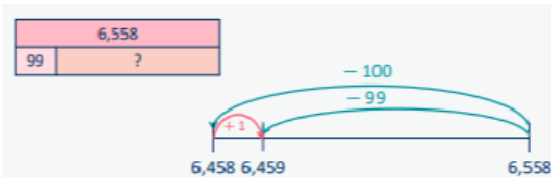
**Subtract using mental strategies**

Subtract 1s, 10s, 100s etc from any number. Use number bonds and related facts.



48,650 - 300 =  
 48,650 - 30,000 =  
 48,650 - 30 =

To subtract ..., I can subtract ... then add ...



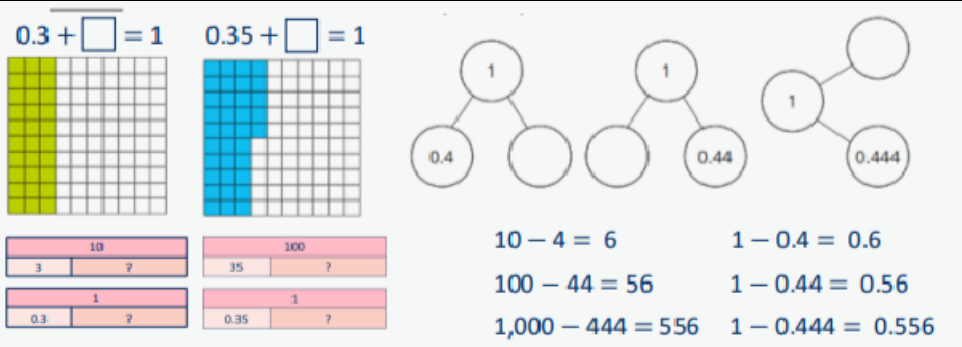
**Subtract decimals with up to 2 decimal places**

Progress from the same number of decimal places to a different number of decimal places and from no exchange to exchange.



**Complements to 1**

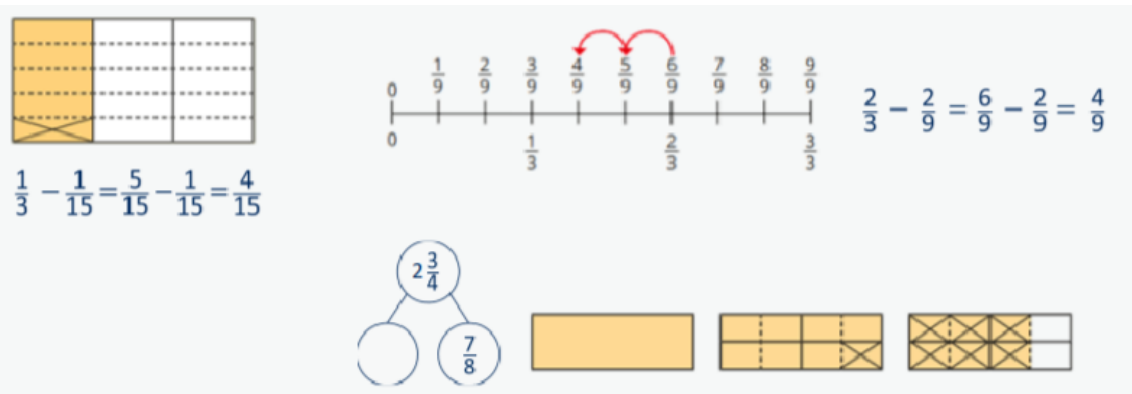
Encourage children to make links with bonds to 10 and complements to 100 and 1,000 when finding a missing part or subtracting from 1



**Subtract fractions with denominators that are a multiple of one another**



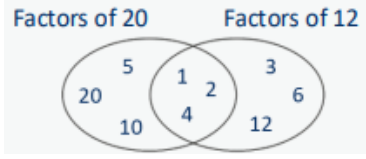
Convert fractions to the same denominator before subtracting. Progress from subtracting fractions within 1 whole to subtracting from a mixed number.


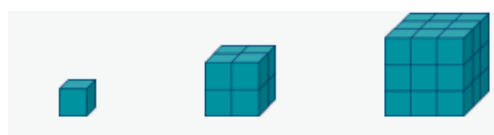
The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.

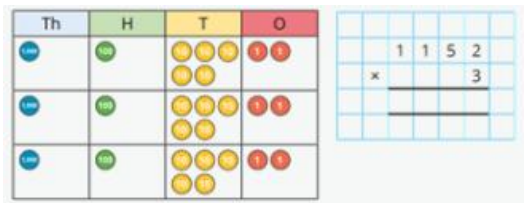


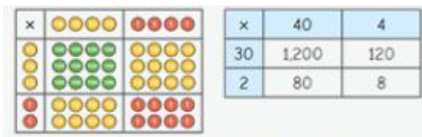
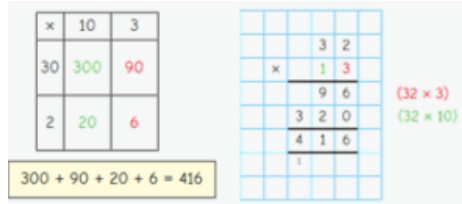
<b>Multiplication</b>	<ul style="list-style-type: none"> <li>Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers</li> <li>Recognise and use square numbers and cube numbers, and the notation for squared (<sup>2</sup>) and cubed (<sup>3</sup>)</li> <li>Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.</li> <li>Multiply numbers mentally drawing upon known facts.</li> <li>Multiply whole numbers and those involving decimals by 10, 100 and 1000</li> <li>Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.</li> </ul>
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<b>Key skills</b>	<b>Key representations</b>
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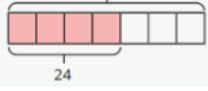
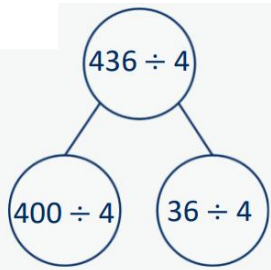
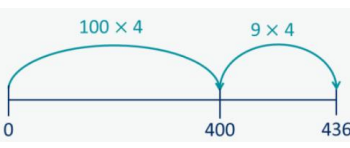
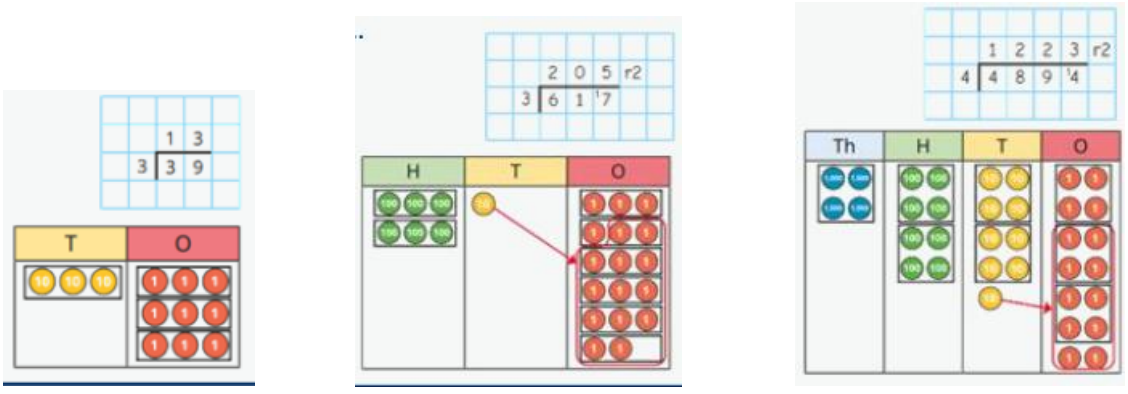
<b>Multiples and factors</b>	<p>... is a multiple of ... because</p> <p>... × ... = ...</p>  <table border="1" data-bbox="375 806 734 918"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	<p>... is a factor of ... because</p> <p>... × ... = ...</p>  <p>1, 2, 4 and 8 are factors of 8</p>	<p>The common factors of ... and ... are ...</p> 
1	2	3	4	5	6	7	8	9	10																								
11	12	13	14	15	16	17	18	19	20																								
21	22	23	24	25	26	27	28	29	30																								

<b>Square and cube numbers</b>	<p>... squared means ... × ...</p>  <p> <math>1 \times 1</math>   <math>2 \times 2</math>   <math>3 \times 3</math>   <math>4 \times 4</math>  <math>1^2 = 1</math>   <math>2^2 = 4</math>   <math>3^2 = 9</math>   <math>4^2 = 16</math> </p>	<p>... cubed means ... × ... × ...</p>  <p> <math>1 \times 1 \times 1</math>   <math>2 \times 2 \times 2</math>   <math>3 \times 3 \times 3</math>  <math>1^3 = 1</math>   <math>2^3 = 8</math>   <math>3^3 = 27</math> </p>
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<b>Multiply numbers up to 4 digits by a 1-digit number</b>	<p>To multiply a 4-digit number by ... , I multiply the ones by ... , the tens by ... , the hundreds by ... and the thousands by ...</p> 
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<b>Multiply numbers up to 4 digits by a 2-digit number</b>	<p>I can partition ... into ... and ...</p>  <p> <math>32 \times 44 = 1,200 + 80 + 120 + 8</math>  <math>32 \times 44 = 1,408</math> </p>	<p>First, I multiply by the ... Then I multiply by the ...</p>  <p><math>300 + 90 + 20 + 6 = 416</math></p>
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<p><b>Multiply by 10, 100 and 1,000</b></p> <p>Some children may overgeneralise that multiplying by a power of 10 always results in adding zeros. This will cause issues later when multiplying decimals.</p>	<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><th>M</th><th>HTh</th><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td></td><td></td><td></td><td></td><td>●●</td><td>●●●</td><td>●●●</td></tr> </table> <p> <math>234 \times 10 = 2,340</math>  <math>234 \times 100 = 23,400</math>  <math>234 \times 1,000 = 234,000</math> </p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th><th>Tth</th><th>Hth</th></tr> <tr><td></td><td></td><td></td><td>●●</td><td>●●●</td><td>●●●</td></tr> </table> <p> <math>2.34 \times 10 = 23.4</math>  <math>2.34 \times 100 = 234</math>  <math>2.34 \times 1,000 = 2,340</math> </p> </div> </div>	M	HTh	TTh	Th	H	T	O					●●	●●●	●●●	Th	H	T	O	Tth	Hth				●●	●●●	●●●
M	HTh	TTh	Th	H	T	O																					
				●●	●●●	●●●																					
Th	H	T	O	Tth	Hth																						
			●●	●●●	●●●																						
<p><b>Mental strategies</b></p> <p>Children continue to use efficient mental strategies such as partitioning and knowledge of factor pairs and related facts to multiply.</p>	<p>The most efficient strategy to calculate ... <math>\times</math> ... is ... To calculate ... <math>\times</math> 12, I can do ... <math>\times</math> ... <math>\times</math> ...</p> <p><b>For example: <math>121 \times 12</math></b></p> <p>I could calculate <math>100 \times 12</math> plus <math>20 \times 12</math> plus <math>1 \times 12</math></p> <p>I could calculate <math>121 \times 10</math> plus <math>121 \times 2</math></p> <p>I could calculate <math>121 \times 6 \times 2</math></p> <p>I could calculate <math>121 \times 4 \times 3</math></p>																										
<p><b>Multiply fractions by a whole number</b></p> <p>Make links with repeated addition.</p> <p>E.g. <math>\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}</math></p>	<p>To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td></td></tr> </table> <p><math>\frac{1}{7} \times 5 = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}</math></p> <table border="1" style="margin-bottom: 5px;"> <tr><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td></tr> </table> <p><math>\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}</math></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p><math>\frac{2}{7} \times 3 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}</math></p> <table border="1" style="margin-bottom: 5px;"> <tr><td><math>\frac{2}{5}</math></td><td><math>\frac{2}{5}</math></td><td><math>\frac{2}{5}</math></td></tr> </table> <p><math>\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}</math></p> </div> </div>	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$					
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$																							
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$																						
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$																									
<p><b>Multiply mixed numbers by a whole number</b></p>	<p>I can partition <math>\frac{2}{3}</math> into <math>\frac{1}{3}</math> and <math>\frac{1}{3}</math></p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr> </table> <p><math>2\frac{2}{3} \times 3</math></p> </div> <div style="text-align: center;"> <p><math>2 \times 3 = 6</math></p> <p><math>\frac{2}{3} \times 3 = \frac{6}{3} = 2</math></p> </div> </div> <p><math>2\frac{2}{3} \times 3 = 6 + 2 = 8</math></p>	■	■	■	■	■																					
■	■	■	■	■																							

<p><b>Find the whole</b></p> <p>Children multiply to find the whole from a given part.</p>	<p>If <math>\frac{1}{5}</math> is ... , then the whole is ... <math>\times</math> ...</p> <p><math>\frac{1}{5}</math> of ___ = 6</p>  <p><math>5 \times 6 = 30</math></p> <p><math>\frac{1}{5}</math> of <b>30</b> = 6</p>	<p>If <math>\frac{1}{4}</math> is ... , then <math>\frac{1}{7}</math> is ... and the whole is ... <math>\times</math> ...</p> <p><math>\frac{4}{7}</math> of ___ = 24</p>  <p><math>\frac{1}{7} = 24 \div 4 = 6</math></p> <p><math>7 \times 6 = 42</math></p> <p><math>\frac{4}{7}</math> of <b>42</b> = 24</p>	
<p><b>Division</b></p>	<ul style="list-style-type: none"> <li>• Divide numbers mentally drawing upon known facts.</li> <li>• Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.</li> <li>• Divide whole numbers and those involving decimals by 10, 100 and 1,000</li> </ul>		
<p><b>Key skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Mental strategies</b></p>	<p>I can partition ... into ... and ... to help me to divide more easily.</p> 	<p>I can show groups of ... on a number line.</p> 	<p>To divide by ..., I can divide by ... and then divide the result by ...</p> <p><math>436 \div 4 = 436 \div 2 \div 2</math></p> <p><math>436 \div 2 = 218</math></p> <p><math>218 \div 2 = 109</math></p>
<p><b>Divide numbers up to 4 digits by a 1-digit number</b></p> <p>The short division method is introduced for the first time.</p>	<p>There are ... groups of ... hundreds/tens/ones/ in ...</p> <p>I can exchange 1 ... for 10 ...</p> 		
<p><b>Divide by 10, 100 and 1,000</b></p> <p>Encourage children to notice that dividing by 100 is the same as</p>	<p>To divide by 10/100/1,000, I move all the digits ... places to the right.</p> <p>... is one-tenth/one-hundredth/one-thousandth the size of ...</p>		

dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.

Th	H	T	O	Tth	Hth
	●	●●	●		

$120 \div 10 = 12$

Th	H	T	O	Tth	Hth
		●	●●	●	

$120 \div 100 = 1.2$

Th	H	T	O	Tth	Hth
			●	●●	●

$120 \div 1,000 = 0.12$

**Fraction of an amount**  
Bar models support children to understand that to find a fraction of an amount, we divide by the denominator and multiply by the numerator.

To find  $\frac{1}{5}$  of ..., I need to divide by ... and multiply by ...

$\frac{1}{5}$  of 20 =                       $\frac{1}{4}$  of 84 =

$\frac{3}{5}$  of 20 =                       $\frac{3}{4}$  of 84 =

If  $\frac{1}{5}$  is ..., then the whole is ...  $\times$  ...

$\frac{1}{5}$  of \_\_\_ = 6

$\frac{4}{7}$  of \_\_\_ = 24

**Year 6**

- Addition**
- Add larger numbers, using the formal written method of columnar addition.
  - Use their knowledge of the order of operations to carry out calculations involving the 4 operations.
  - Calculate intervals across zero.
  - Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

**Key skill**      **Key representations**

**Add integers up to 10 million**

Encourage children to estimate and use inverse operations to check answers to calculations.

**Add decimals with up to 3 decimal places**

Progress to numbers with digits in different place value columns.

Encourage children to check that they have lined up the columns correctly.

I do/do not need to make an exchange because ...

**Order of operations**

Calculations in brackets should be done first.

Multiplication and division should be performed before addition and subtraction.

\*When no brackets are shown and the operations have the same priority, work left to right.

... has greater priority than ..., so the first part of the calculation I need to do is ...

**Negative numbers**

Children add to negative numbers and carry out calculations which cross 0

... plus ... is equal to ...

$-3 + 5 = 2$

$-11 + 16 = 5$

The difference between  $-5$  and  $-1$  is  $4$

The difference between  $-5$  and  $5$  is  $10$

**Add fractions**

Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.

The denominator has been multiplied by ..., so the numerator needs to be multiplied by ...

The lowest common multiple of ... and ... is ...

$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

...is made up of ... wholes and ...

**Subtraction**

- Subtract larger numbers, using the formal written methods of columnar subtraction.
- Use their knowledge of the order of operations to carry out calculations involving the 4 operations.
- Calculate intervals across zero.
- Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

**Key skills**      **Key representations**

**Subtract integers up to 10 million**

Encourage children to estimate and use inverse operations to check answers to calculations.

4,604		
2,354	750	?

**Subtract decimals with up to 3 decimal places**

Progress from the same number of decimal and whole number places to a different number of decimal and whole number places.

I do/do not need to make an exchange because ...

**Order of operations**

Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...

**Negative numbers**

Children subtract from positive and negative numbers and calculate intervals across 0

... minus ... is equal to ...

$-1 - 4 = -5$

$1 - 4 = -3$

The difference between  $-5$  and  $-1$  is 4

The difference between 5 and  $-5$  is 10

**Subtract fractions**

Convert fractions to the same denominator before subtracting. Progress from fractions where one

The denominator has been multiplied by ..., so the numerator needs to be multiplied by...	The lowest common multiple of ... and ... is ...	... is made up of ... wholes and ...
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<p>denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.</p>	$\frac{2}{3} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9}$	$\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$	$2\frac{3}{4} - 1\frac{1}{8} = 1\frac{5}{8}$
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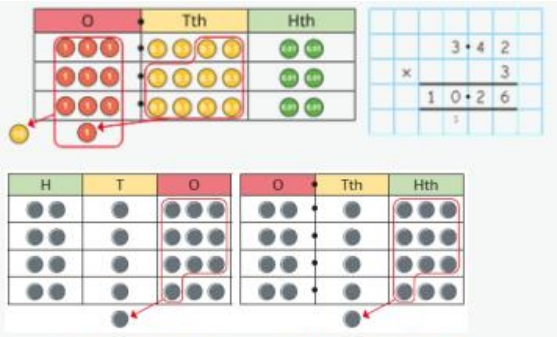
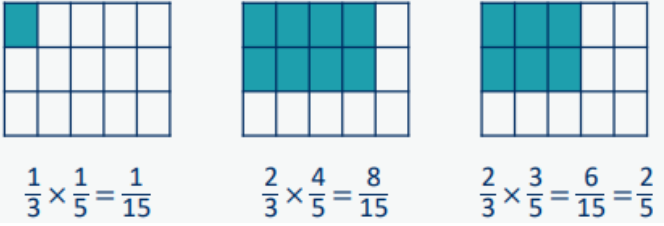
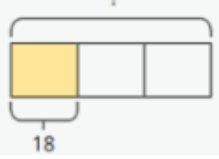
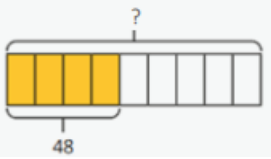
<p><b>Multiplication</b></p>	<ul style="list-style-type: none"> <li>Identify common factors and common multiples.</li> <li>Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</li> <li>Multiply numbers by 10, 100 and 1,000</li> <li>Multiply one-digit numbers with up to two decimal places by whole numbers.</li> <li>Use their knowledge of the order of operations to carry out calculations involving the 4 operations.</li> <li>Multiply simple pairs of proper fractions, writing the answer in its simplest form.</li> <li>Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.</li> <li>Solve problems involving the calculation of percentages.</li> </ul>
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<p><b>Key skills</b></p>	<p><b>Key representations</b></p>
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<p><b>Multiply numbers up to 4 digits by a 2-digit number</b></p>	<p>To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.</p>	
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<p><b>Multiply by 10, 100 and 1,000</b></p> <p>Some children may overgeneralise that multiplying by a power of 10 always results in adding zeros.</p>	<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>M</td><td>HTh</td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td> <td>Th</td><td>H</td><td>T</td><td>O</td><td>Tth</td><td>Hth</td><td>Thth</td> </tr> <tr> <td></td><td></td><td></td><td></td><td>●●</td><td>●●</td><td>●●</td> <td></td><td></td><td></td><td>●</td><td>●●</td><td>●●</td><td>●●</td> </tr> </table> <p> <math>234 \times 10 = 2,340</math>  <math>234 \times 100 = 23,400</math>  <math>234 \times 1,000 = 234,000</math> </p> <p> <math>0.234 \times 10 = 2.34</math>  <math>0.234 \times 100 = 23.4</math>  <math>0.234 \times 1,000 = 234</math> </p>	M	HTh	TTh	Th	H	T	O	Th	H	T	O	Tth	Hth	Thth					●●	●●	●●				●	●●	●●	●●
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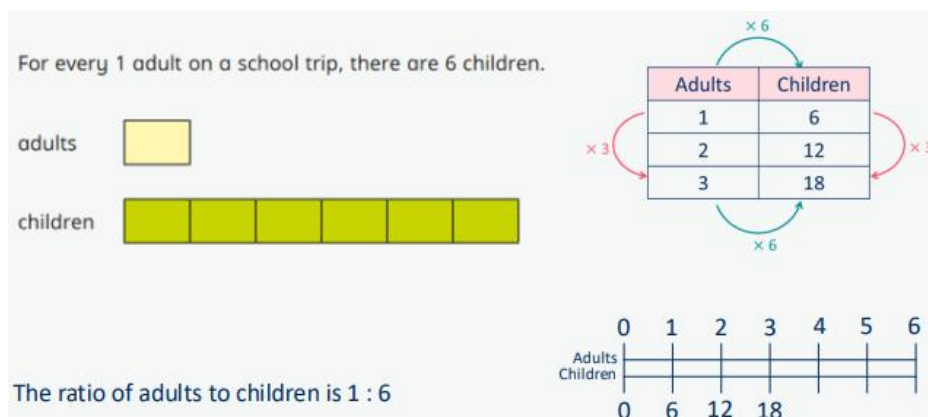
<p><b>Order of operations</b></p> <p>Calculations in brackets should be done first. Multiplication and division should be performed before</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> <p><math>(3 + 4) \times 2 = 14</math></p> </div> <div style="text-align: center;"> <p><math>3 + 4 \times 2 = 11</math></p> </div> <div style="text-align: center;"> <p><math>3 + 4^2 = 19</math></p> </div> </div>
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<p>addition and subtraction.</p>																																		
<p><b>Multiply decimals by integers</b></p> <p>This is the first time children multiply decimals by numbers other than 10, 100 or 1,000 Encourage them to make links with known facts and whole number multiplication.</p>	<p>I know that <math>... \times ... = ...</math>, so I also know that <math>... \times ... = ...</math></p>  <p><math>6 \times 2 = 12</math>      <math>6 \times 0.2 = 1.2</math></p>	<p>I need to exchange 10 ... for 1 ...</p>  <p><math>213 \times 4 = 852</math>      <math>2.13 \times 4 = 8.52</math></p>																																
<p><b>Multiply fractions by fractions</b></p> <p>Encourage children to give answers in their simplest form.</p>	<p>When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.</p>  <p><math>\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}</math>      <math>\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}</math>      <math>\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}</math></p>																																	
<p><b>Find the whole</b></p> <p>Children multiply to find the whole from a given part.</p>	<p>If <math>\frac{1}{3}</math> is ... , then the whole is <math>... \times ...</math></p>  <p><math>\frac{1}{3}</math> of <math>?</math> = 18</p> <p><math>18 \times 3 = 54</math></p> <p><math>\frac{1}{3}</math> of <b>54</b> = 18</p>	<p>If <math>\frac{4}{9}</math> is ... , then <math>\frac{1}{9}</math> is ... and the whole is <math>... \times ...</math></p>  <p><math>\frac{4}{9}</math> of <math>?</math> = 48</p> <p><math>\frac{1}{9} = 48 \div 4 = 12</math></p> <p><math>9 \times 12 = 108</math></p> <p><math>\frac{4}{9}</math> of <b>108</b> = 48</p>																																
<p><b>Calculate percentages</b></p> <p>Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</p>	<p>There are ... lots of ... % in 100%</p> <p>To find ... %, I need to divide by ...</p> <table border="1" data-bbox="383 1612 837 1713"> <tr><td colspan="4">100%</td></tr> <tr><td colspan="2">50%</td><td colspan="2">50%</td></tr> <tr><td>25%</td><td>25%</td><td>25%</td><td>25%</td></tr> </table> <p>50% of ... = <math>... \div 2</math> 25% of ... = <math>... \div 4</math></p>	100%				50%		50%		25%	25%	25%	25%	<p>... % is made up of ... %, and ... %</p> <table border="1" data-bbox="981 1579 1524 1646"> <tr><td colspan="10">100%</td></tr> <tr><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td></tr> </table> <p>To find 30%, I can find 10% and then multiply it by 3 To find 23%, I can use <math>10\% \times 2</math> and <math>1\% \times 3</math> To find 99%, I can find 1%, then subtract from 100%</p>	100%										10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
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10%	10%	10%	10%	10%	10%	10%	10%	10%	10%																									

**Calculations involving ratio**

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...



**Division**

- Perform mental calculations, including with mixed operations and large numbers.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Divide numbers by 10, 100 and 1,000 giving answers up to three decimal places.
- Use written division methods in cases where the answer has up to two decimal places.
- Associate a fraction with division and calculate decimal fraction equivalents.
- Divide proper fractions by whole numbers [for example,  $1 \div 2 = \frac{1}{2}$ ]
- Solve problems involving the calculation of percentages

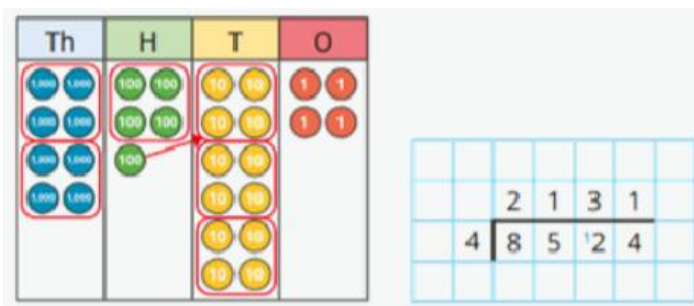
**Key skills**



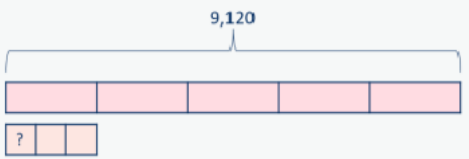
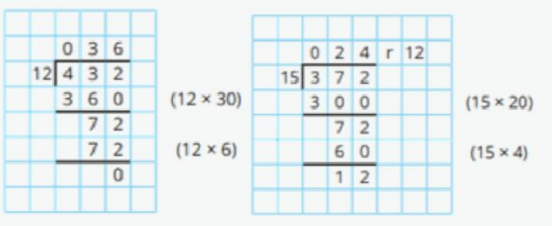
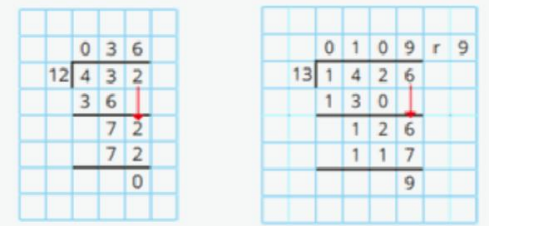
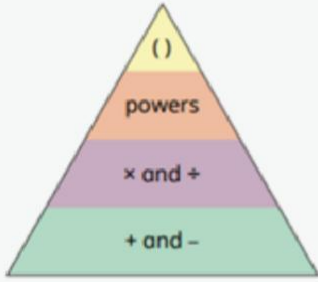
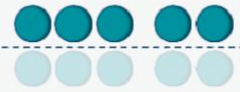


**Key representations**


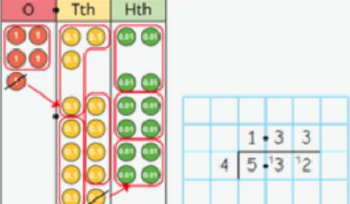
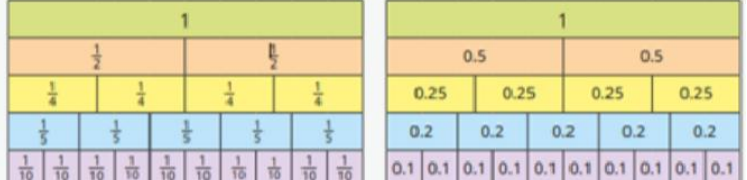
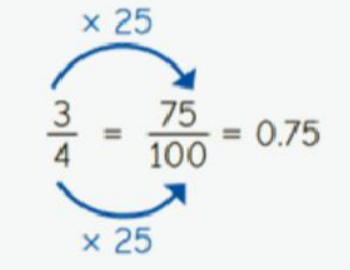
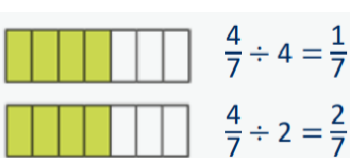

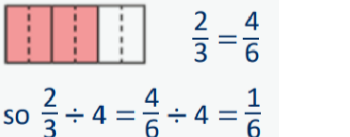
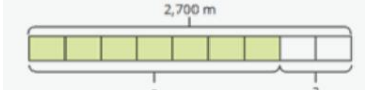
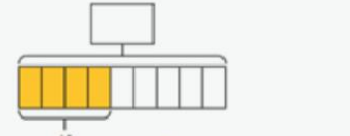
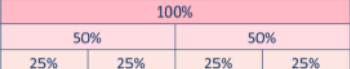
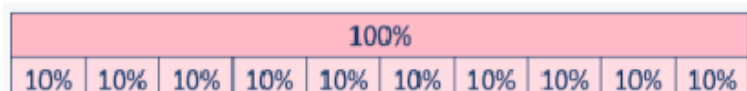
**Short division**

Encourage children to interpret remainders in context, for example knowing that “remainder 1” could mean complete boxes with 1 left over so 5 boxes will be needed.

There are ... groups of ... hundreds/tens/ones/ in ...  
I can exchange 1 ... for 10 ...



<p><b>Mental strategies</b></p> <p>Include partitioning and number line strategies outlined in Y5 as well as division using factors.</p>	<p>To divide by ... , I can first divide by ... and then divide the answer by ...</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <math>240 \div 60 = 240 \div 10 \div 6</math>    <math>480 \div 24 = 480 \div 4 \div 6</math>   </div> <div style="text-align: center;"> <math>9,120 \div 15 = 9,120 \div 5 \div 3</math>   </div> </div>	
<p><b>Long division</b></p> <p>The long division method is introduced for the first time. Two alternative methods are shown.</p>	<p>Method 1</p> 	<p>Method 2</p> 
<p><b>Order of operations</b></p> <p>Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> <div style="display: flex; align-items: center;"> <div style="text-align: center; margin-right: 20px;">  </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">   <math>(6 + 4) \div 2 = 5</math> </div> <div style="text-align: center;">   <math>6 + 4 \div 2 = 8</math> </div> </div> </div>	
<p><b>Divide by 10, 100 and 1,000</b></p> <p>Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.</p>	<p>To divide by ... , I move the digits ... places to the right.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">  </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <math>312 \div 10 = 31.2</math>  <math>312 \div 100 = 3.12</math>  <math>312 \div 1,000 = 0.312</math> </div> <div style="text-align: center;"> <math>906 \div 10 = 90.6</math>  <math>906 \div 100 = 9.06</math>  <math>906 \div 1,000 = 0.906</math> </div> </div> </div>	
<p><b>Divide decimals by integers</b></p> <p>This is the first time children divide decimals by numbers</p>	<p>I know that ... <math>\div</math> ... = ..., so I also know that ... <math>\div</math> ... = ...</p>	<p>I need to exchange 1 ... for 10 ...</p>

<p>other than 10, 100 or 1,000</p>	 <p><math>39 \div 3 = 13</math>      <math>3.9 \div 3 = 1.3</math>      <math>0.39 \div 3 = 0.13</math></p>		
<p><b>Decimal and fraction equivalents</b></p>	<p>The fraction ... is equivalent to the decimal ...</p>  <p><math>\frac{1}{5} = 0.2</math>      <math>\frac{2}{5} = 0.4</math>      <math>\frac{3}{5} = 0.6</math></p>	<p><input type="text"/> is equal to <math>\frac{\text{input}}{100}</math></p>  <p><math>\frac{3}{4} = \frac{75}{100} = 0.75</math></p>	
<p><b>Divide a fraction by an integer</b></p> <p>This is the first time children divide fractions by an integer.</p>	<p>... ones divided by 2 is ... ones so ... sevenths divided by 2 is ... sevenths.</p>  <p><math>\frac{4}{7} \div 4 = \frac{1}{7}</math> <math>\frac{4}{7} \div 2 = \frac{2}{7}</math></p>	<p>I am dividing by ..., so I can split each part into ... equal parts.</p>  <p><math>\frac{1}{3} \div 2 = \frac{1}{6}</math></p>	<p>... is equivalent to ... so ... <math>\div</math> ... = ... <math>\div</math> ...</p>  <p>so <math>\frac{2}{3} \div 4 = \frac{4}{6} \div 4 = \frac{1}{6}</math></p>
<p><b>Fraction of an amount</b></p> <p>Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.</p>	<p>To find <math>\frac{1}{\square}</math> I divide by ...</p> <p><math>\frac{1}{2}</math> of 36 = <math>36 \div 2</math> <math>\frac{1}{12}</math> of 36 = <math>36 \div 12</math></p>	<p>If <math>\frac{1}{\square}</math> is equal to ..., then <math>\frac{\square}{\square}</math> are equal to ...</p>  <p><math>\frac{7}{9}</math> of 2,700 = <math>\frac{1}{9}</math> of 2,700 <math>\times 7</math></p>	<p>If <math>\frac{\square}{\square}</math> is equal to ..., then the whole is equal to ...</p>  <p><math>\frac{4}{9}</math> of ___ = 48</p>
<p><b>Calculate percentages</b></p> <p>Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</p>	<p>There are ... lots of ... % in 100%</p> <p>To find ... %, I need to divide by ...</p>  <p>50% of ... = ... <math>\div 2</math> 25% of ... = ... <math>\div 4</math></p>	<p>... % is made up of ... %, and ... %</p>  <p>To find 30%, I can find 10% and then multiply it by 3 To find 23%, I can use 10% <math>\times 2</math> and 1% <math>\times 3</math> To find 99%, I can find 1%, then subtract from 100%</p>	

**Calculations involving ratio**

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships

For every ... , there are ...

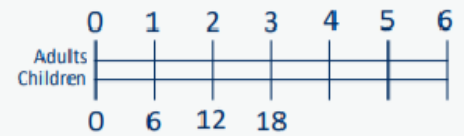
For every 6 children on a school trip, there is 1 adult.



Adults	Children
1	6
2	12
3	18

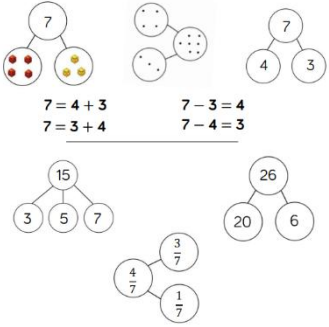

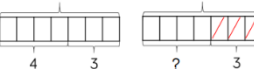
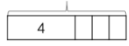






Annotations: A blue arrow from 6 to 18 is labeled  $\div 6$ . A red arrow from 1 to 2 is labeled  $\times 2$ . A red arrow from 6 to 12 is labeled  $\times 2$ . A red arrow from 12 to 18 is labeled  $\times 1.5$ .

The ratio of children to adults is 6 : 1

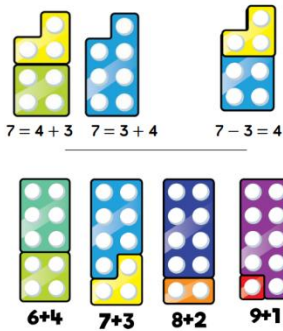


# Overview of different models and images that can support the teaching of mathematical concepts.

These provide explanations of the benefits of using the models and show the links between different operations

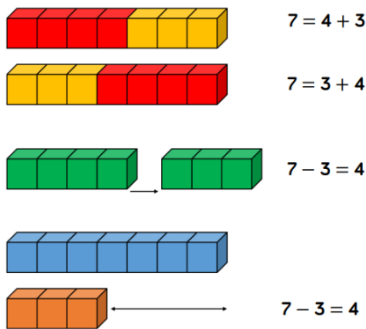
<b>Addition and Subtraction</b>	
Part-Whole Model	
 <p>7 = 4 + 3 7 = 3 + 4</p> <p>7 - 3 = 4 7 - 4 = 3</p> <p>15 = 3 + 5 + 7</p> <p>26 = 20 + 6</p> <p><math>\frac{3}{7}</math> <math>\frac{4}{7}</math> <math>\frac{1}{7}</math></p>	<p>This part-whole model supports children in their understanding of aggregation and partitioning. When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total. When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part. Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns. In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.</p>
Bar Model (single)	
<p><b>Concrete</b> </p> <p><b>Discrete</b> </p> <p><b>Combination</b> </p> <p><b>Continuous</b> </p> <p></p>	<p>The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure. Cubes and counters can be used in a line as a concrete representation of the bar model. Discrete bar models are a good starting point with smaller numbers. Each box represents one whole. The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model. Continuous bar models can be used to represent a range of values. Each rectangle represents a number. The question mark indicates the value to be found. In KS2, children can use bar models to represent larger numbers, decimals and fractions.</p>
Bar Model (Multiple)	
<p><b>Discrete</b></p> <p> <math>7 + 3 = 10</math></p> <p> <math>7 - 3 = 4</math></p> <p><b>Continuous</b></p> <p> <math>7 - 3 = 4</math></p> <p> <math>2,394 - 1,014 = 1,380</math></p>	<p>The multiple bar model is a good way to compare quantities whilst still unpicking the structure. Two or more bars can be drawn, with a bracket labelling the whole positioned on the right-hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers. Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference. When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.</p>

## Numicon



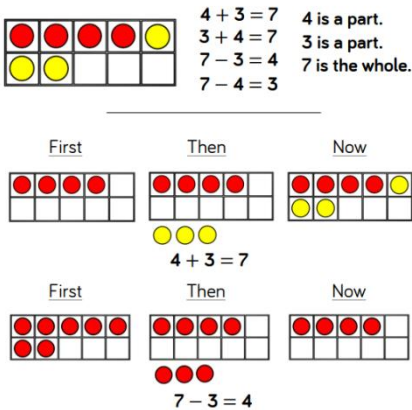
Numicon can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds. When adding numbers, children can see how the parts come together making a whole. As children use numicon more often, they can start to subitise the total due to their familiarity with the shape of each number. When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes. Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

## Cubes



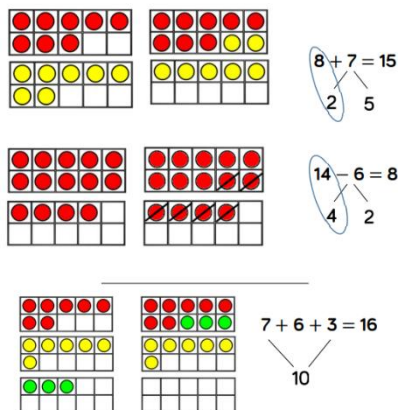
Cubes can be useful to support children with the addition and subtraction of one-digit numbers. When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole. When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away. Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers. Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

## Ten Frames (within 10)



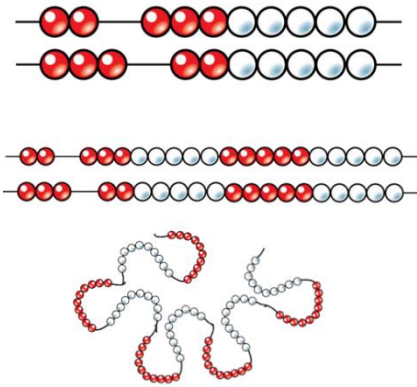
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction. Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number. Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

## Ten Frames (within 20)



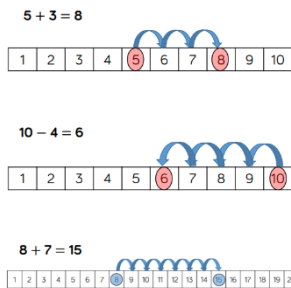
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition. When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction. When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

## Bead strings



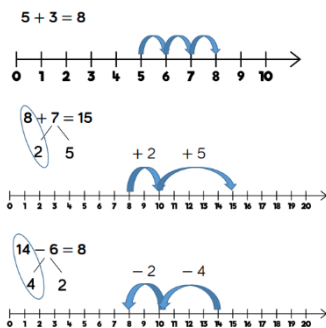
Different sizes of bead strings can support children at different stages of addition and subtraction. Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g.  $2 + 8 = 10$ , move one bead,  $3 + 7 = 10$ . Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20. Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

## Number Tracks



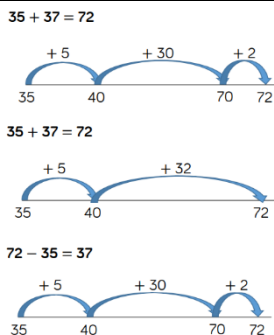
Number tracks are useful to support children in their understanding of augmentation and reduction. When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total. When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers. Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back. Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

## Number lines (labelled)



Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction. Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track. Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part. Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

## Number lines (blank)



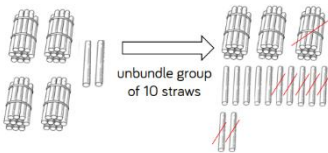
Blank number lines provide children with a structure to add and subtract numbers in smaller parts. Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately. Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number. Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

## Straws

$$7 + 6 = 13$$

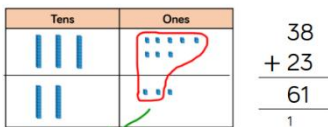


$$42 - 17 = 25$$

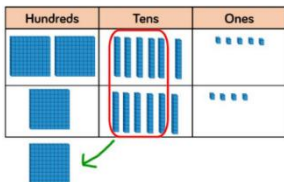


Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers. Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws. When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total. When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones. Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

## Base 10/Dienes (addition)



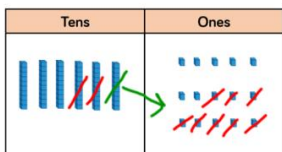
$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$



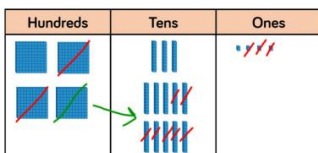
$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can make clear links between the written method and the model. Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use. When adding, always start with the smallest place value column. Some questions to support children could be: How many ones are there altogether? Can we make an exchange? (Yes or No) How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

## Base 10/Dienes (subtraction)



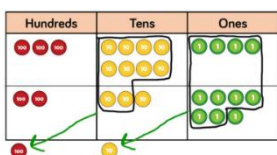
$$\begin{array}{r} 5 \cancel{1} \\ 65 \\ - 28 \\ \hline 37 \end{array}$$



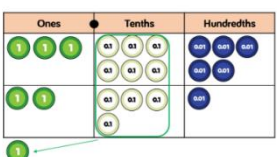
$$\begin{array}{r} 3 \cancel{1} \\ 435 \\ - 273 \\ \hline 262 \end{array}$$

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model. Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently. This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

## Place Value Counters (addition)



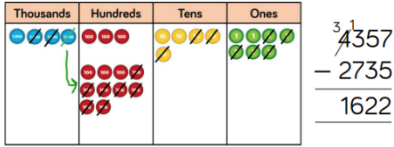
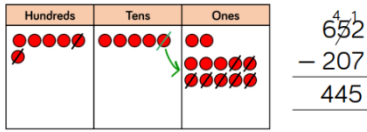
$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model. Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns. When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

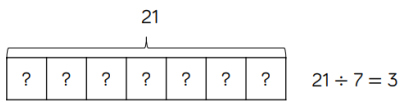
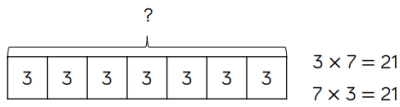
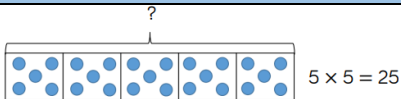
## Place Value Counters (Subtraction)



Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model. Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns. When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

## Multiplication and Division

### Bar Model

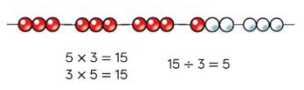
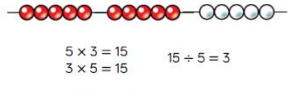
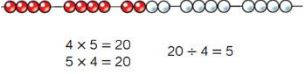


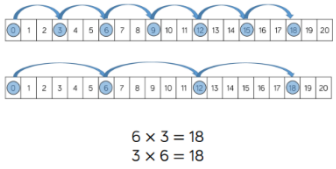
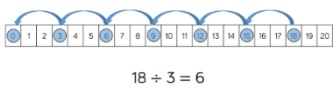
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication. Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups. It is important when solving word problems that the bar model represents the problem. Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there? The multiple bar model provides an opportunity to compare the groups.

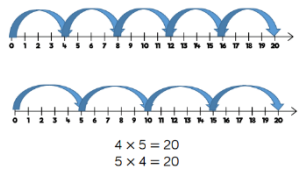
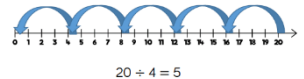
### Numicon

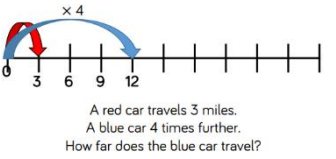
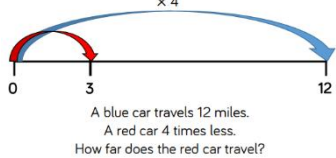


Numicon can support children's understanding of multiplication as repeated addition. Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the numicon in multiplication can support children in discovering patterns of multiplication e.g., odd x odd = even, odd x even = odd, even x even = even. When dividing, numicon support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

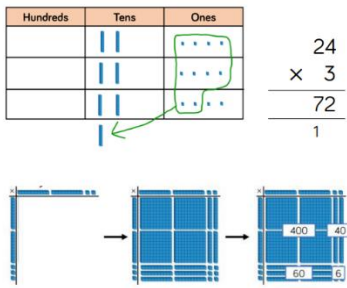
Bead Strings	
 <p> <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math>  <math>15 \div 3 = 5</math> </p>	<p>Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently. Encourage children to count in multiples as they build the number e.g., 4, 8, 12, 16, 20. Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count. When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g., 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.</p>
 <p> <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math>  <math>15 \div 5 = 3</math> </p>	
 <p> <math>4 \times 5 = 20</math>  <math>5 \times 4 = 20</math>  <math>20 \div 4 = 5</math> </p>	

Number Tracks	
 <p> <math>6 \times 3 = 18</math>  <math>3 \times 6 = 18</math> </p>	<p>Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting. When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers. When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division. Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.</p>
 <p> <math>18 \div 3 = 6</math> </p>	

Number Lines (labelled)	
 <p> <math>4 \times 5 = 20</math>  <math>5 \times 4 = 20</math> </p>	<p>Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications. When multiplying, children start at 0 and then count on to find the product of the numbers. When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division. Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.</p>
 <p> <math>20 \div 4 = 5</math> </p>	

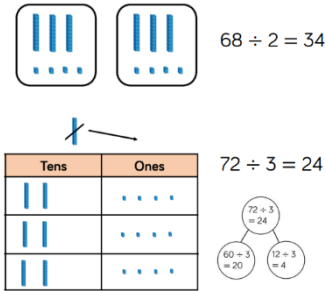
Number Lines (blank)	
 <p> <math>\times 4</math>      A red car travels 3 miles.      A blue car 4 times further.      How far does the blue car travel?   </p>	<p>Children can use blank number lines to represent scaling as multiplication or division. Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems. Blank number lines without intervals can also be used for children to represent scaling.</p>
 <p> <math>\times 4</math>      A blue car travels 12 miles.      A red car 4 times less.      How far does the red car travel?   </p>	

Base 10/Dienes (multiplication)



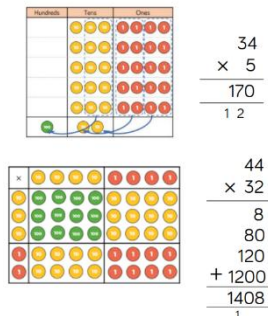
Using Base 10 or Dienes is an effective way to support children’s understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match. As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)



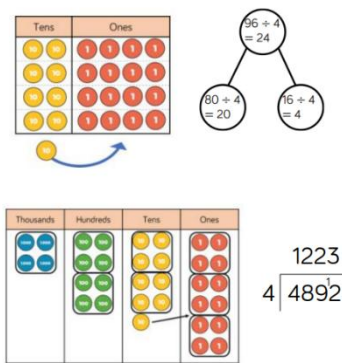
Using Base 10 or Dienes is an effective way to support children’s understanding of division. When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g., by drawing circles or by rows on a place value grid. When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g., one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)



Using place value counters is an effective way to support children’s understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match. As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic. Place value counters also support the area model of multiplication well. Children can see how to multiply 2- digit numbers by 2-digit numbers.

Place Value Counters (division)



Using place value counters is an effective way to support children’s understanding of division. When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g., exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking. Place value counters also support children’s understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g., exchange one hundred for ten tens.

# Glossary

Addition and subtraction	
Addend	A number to be added to another
Aggregation	combining two or more quantities or measures to find a total
Augmentation	Increasing a quantity or measure by another quantity.
Commutative	Numbers can be added in any order.
Complement	in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000
Difference	The numerical difference between two numbers is found by comparing the quantity in each group.
Exchange	Change a number or expression for another of an equal value
Minuend	A quantity or number from which another is subtracted.
Partitioning	Splitting a number into its component parts.
Reduction	Subtraction as take away.
Subitise	Instantly recognise the number of objects in a small group without needing to count
Subtrahend	A number to be subtracted from another.
Sum	The result of an addition
Total	The aggregate or the sum found by addition.

Multiplication and division	
Array	An ordered collection of counters, cubes or other item in rows and columns.
Commutative	Numbers can be multiplied in any order.
Dividend	In division, the number that is divided
Divisor	In division, the number by which another is divided.
Exchange	Change a number or expression for another of an equal value
Factor	A number that multiplies with another to make a product.
Partitioning	Splitting a number into its component parts.
Product	The result of multiplying one number by another.
Quotient	The result of a division
Remainder	The amount left over after a division when the divisor is not a factor of the dividend
Scaling	Enlarging or reducing a number by a given amount, called the scale factor